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Vol. 218 **JULY, 1934** **No. 1**

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Vol. 218 JULY, 1934 No. 1

HEAVY HYDROGEN—A NEW RESEARCH TOOL *

BY

HUGH S. TAYLOR, D.Sc.,

David B. Jones Professor of Chemistry, Princeton University.

The discovery of a second hydrogen isotope, deuterium, of mass 2,[1] the detection of its enrichment in water by electrolytic generation of hydrogen and oxygen [2] and the demonstration that prolonged electrolysis could be used to isolate the pure deuterium free from the lighter isotope [3] were events of profound significance to all those whose research interests involved either hydrogen or its compounds. The wide distribution of this element in nature implied a correspondingly wide distribution of research interest. In the Department of Chemistry at Princeton University this interest was intensified by reason of the extensive program of research conducted over a period of fifteen years on atomic hydrogen and on the interaction of hydrogen gas with surfaces which were known to have an accelerating influence on processes in which hydrogen was one of the reacting constituents. In the second place, in the theoretical quantum mechanical approach to the problem of chemical reaction velocities, a subject of major interest at Princeton, the fundamental developments in this field, initiated by Heitler and London, extended by Eyring and

* Presented at the Stated Meeting held Wednesday, March 21, 1934.

[1] Urey, Brickwedde and Murphy, *Phys. Rev.*, **39**, 164, 864 (1931); **40**, 1 (1932).

[2] Washburn and Urey, *Proc. Nat. Acad. Sci.*, **18**, 496 (1932).

[3] Lewis, *J. Am. Chem. Soc.*, **55**, 1297 (1933); Lewis and Macdonald, *J. Chem. Phys.*, **1**, 341 (1933); *J. Am. Chem. Soc.*, **55**, 3058 (1933).

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

Polanyi,[4] had been largely concerned with the interaction of hydrogen with other simple molecules. In these developments the importance attaching to the zero-point energies of molecules in determining the magnitude of activation energies had been emphasized. It was therefore natural that Eyring [5] should have called attention to the added importance of the zero-point energy in determining the relative rates of reaction of two isotopes such as those of hydrogen and that experimental test of his predictions should have been sought. Hence it was that, prior to the observation by Lewis of the efficiency of the electrolytic method of separation, experiments at Princeton were directed to the separation of the isotopes based on the fractional desorption of hydrogen from solid surfaces. While these experiments actually yielded a separation,[6] it soon became evident that the electrolytic method of preparation of deuterium was the most practical.

The Preparation of Pure Deuterium Oxide.—The first experiments at Princeton on the electrolytic concentration of deuterium were conducted by Taylor and Eyring.[7] The method which they employed, with improvements and minor modifications, is still in use and a plant with a daily capacity of about 3 grams of pure deuterium oxide is in continuous service. In the first stage of the process, approximately 15 gallons of 0.5 N sodium hydroxide are electrolyzed daily in a battery of 960 cells, each cell carrying a mean current of about 9 amperes. The cells are tall cylindrical vessels each of 200 cc. capacity standing in baths of cold running water, the electrodes being made from a strip of iron and one of nickel, welded together and bent twice at right angles, the iron to serve as cathode in one cell, the nickel as anode in the neighboring cell. The current is delivered at a potential of 110 volts to units of 40 cells in series. Electrolysis is continued until the volume of electrolyte is reduced to one-fifth when the residue is removed, distilled to remove electrolyte and the distillate passed to the second stage of electrolysis. This is

[4] Eyring and Polanyi, *Z. physik. Chem.*, **12B**, 279 (1931).

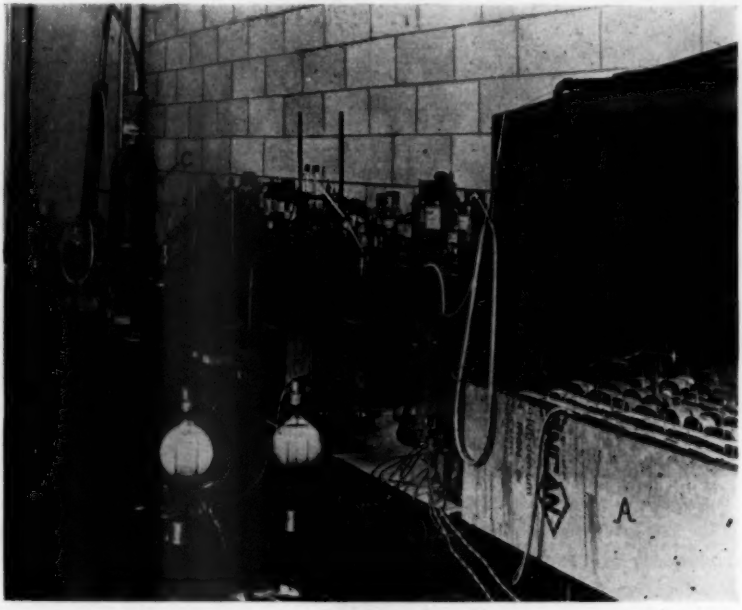
[5] Eyring, *Proc. Nat. Acad. Sci.*, **19**, 78 (1933); *J. Chem. Phys.*, **1**, 345 (1933).

[6] Taylor, Gould and Bleakney, *Phys. Rev.*, **43**, 456 (1933).

[7] Taylor and Eyring, *Proc. Am. Phil. Soc.*, **72**, 255 (1933); Taylor, Eyring and Frost, *J. Chem. Phys.*, **1**, 823 (1933).

entirely similar to the first except that the unit consists of 160 cells only, and handles only one-fifth the volume of liquor, again 0.5 N as to NaOH at the outset. Starting with liquor from commercial electrolytic hydrogen-oxygen generators containing 1 part deuterium to 1600 parts of light hydrogen, the first electrolysis yields liquor of 0.25 per cent. deuterium oxide concentration, the second a 1 per cent. deuterium oxide liquor. In Fig. 1, the second stage unit is shown at the right

FIG. 1.



General View of Electrolytic Concentration of Heavy Water.

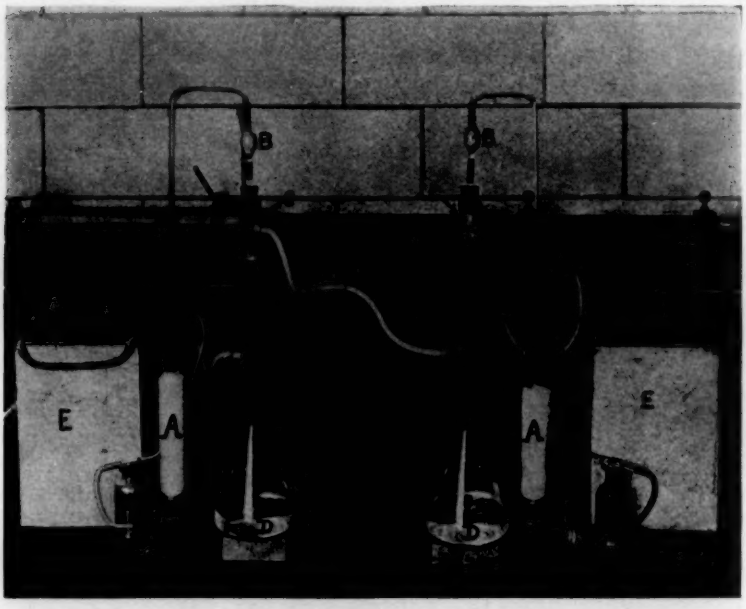
Tank *A* to right contains 160 units for the second electrolysis. Tank *B*, a smaller unit with recovery of hydrogen for more concentrated solutions. The still for distillation of alkali liquors is shown at *C*.

and a copper still for removing the excess electrolyte is shown at the left.

From the third stage onwards a modified form of electrolysis is employed in which the evolved hydrogen and oxygen gases are recovered as water and passed back to a preceding unit. In these stages a closed form of cell is used, the mixed gases are collected, pass by a mercury-sealed blow-off through

a tower for the removal of spray, thence through an explosion trap filled with fine sand to a pyrex jet at which the gases are burned in a silica tube externally cooled by running water, the condensate being collected in a receiver as shown in Fig. 2.

FIG. 2.



Electrolysis with Recovery of Hydrogen and Oxygen by Combustion and Condensation.

The hydrogen and oxygen are freed from spray in the two towers *A* containing absorbent cotton, pass through the explosion traps *B* and are burned at a pyrex jet *C*. The water is condensed and collected in *D*. The enriched residue remains in the vessels in the tanks *E*.

For the third and fourth stages the electrolyses are conducted until the residue from the fourth electrolysis amounts to 1500 cc. with a concentration of 13 per cent. deuterium oxide. This is then electrolyzed in the fifth stage to a residual 240 cc. and a concentration of approximately 40 per cent. D_2O. In a still smaller unit, this 240 cc. is electrolyzed to 45 cc. when the deuterium oxide content is now 95 per cent. The seventh electrolysis will yield a residual 15 cc. of 100 per cent. D_2O. At each stage, the original solution is 0.5 N as to NaOH. The recovered water at each stage is passed back to the electrolytic stage immediately preceding so that from

the third stage onwards the only losses of deuterium are those incidental to the handling of the material and the operations performed. This probably amounts to about 10 per cent. There is some evidence that the efficiency of fractionation is a function of the concentration of the liquor. In good operation a fractionation factor of five can readily be maintained; that is to say, the concentration ratio of light hydrogen to deuterium in the evolved gas is approximately five times that present in the electrolyte from which it is produced.

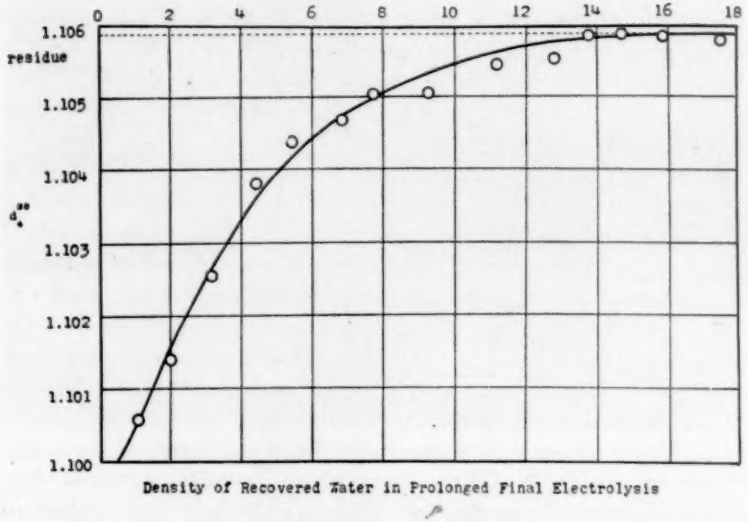
Properties of Heavy Water.—Experience gained in the production of over 100 cubic centimeters of residual heavy water from the seventh stage of the electrolysis has demonstrated that the liquid so received has a density, $D_{25}^{25} = 1.1079$ in contrast to the earlier accepted value of Lewis and Macdonald,[8] $D_{25}^{25} = 1.1056$, which they gave for water containing less than 0.01 per cent. light hydrogen. Since the freezing point of our water of maximum density, 3.82° C., is in best agreement with Lewis and Macdonald's value of 3.8° C., we believe that the difference in density is not to be ascribed to any major difference in purity of the two products but may be attributed to the experimental difficulties inherent in the earlier density determination on a quantity so small as 0.2 cc. Many determinations on variously sized samples aggregating in all 100 cc. have abundantly confirmed the higher value of the density. Further prolonged electrolysis of 25 cc. of water of maximum density in a carefully dried cell using 2 per cent. sodium deuteride, NaOD as electrolyte, failed to produce any further density increase, so that we assign the value of $D_{25}^{25} = 1.1079$ to pure deuterium oxide, D_2O.[9] Figure 3 records the results of such an electrolysis. It will be noted that, in spite of the precautions taken, the first water recovered from such a preparation was only 95 per cent. heavy water. This illustrates the extreme readiness with which pure deuterium oxide takes up moisture from the air and from the containing vessels. It must be handled like any other anhydrous liquid. A sample of density $D_4^{20} = 1.1038$ when exposed to the atmosphere overnight showed a density decrease to 1.1016. Corresponding to the difference in density between

[8] Lewis and Macdonald, *J. Am. Chem. Soc.*, **55**, 3058 (1933).

[9] Taylor and Selwood, *J. Am. Chem. Soc.*, **56**, 998 (1934).

our best preparations and those of others, we also record a greater difference in the refractive indices of heavy and light water. We find this difference to be − 0.00462 at 20° C. for the D-sodium line. The viscosity of deuterium oxide is greater than that of water, the viscosity ratio at 20° C. being 1.249,[10] the value for deuterium oxide being $\eta = 12.6$ millipoises. On the other hand, Selwood and Frost [11] showed that the surface tension of heavy water was less than that of ordinary water. As Bernal has pointed out,[12] these two phe-

FIG. 3.



nomena, both associated with polymerization, go in different directions. The large viscosity of heavy water is a rotary not a translatory viscosity. The heavy water molecule is a slower rotator than the ordinary water molecule; hence heavy water is more ice-like at a given temperature than ordinary water, with a maximum density at 11.6° instead of at 4° C. Hence, also, it has a density somewhat less than would be calculated on the basis of equal molecular volumes of the two species (20/18 = 1.1111). The surface tension depends on the

[10] Lewis and Macdonald, *J. Am. Chem. Soc.*, **55**, 4730 (1933).
[11] Selwood and Frost, *J. Am. Chem. Soc.*, **55**, 4335 (1933).
[12] Bernal, *Proc. Roy. Soc.*, London, **144A**, 24 (1934).

force between the molecules and is not directly affected by the rotation, and so a somewhat smaller value would be expected. Eyring and Topley [13] have developed a theoretical approach to the properties of heavy water along analogous but somewhat different lines.

The Analysis of Deuterium-Containing Mixtures.—To prosecute researches involving deuterium-containing gases it is necessary to possess an analytical method capable of following changes which may occur during the investigations. The method must be capable of distinguishing between the three molecular species possible with light and heavy hydrogen namely H_2, HD and D_2. Moreover, the instrument when in use must not cause reaction among these molecules. Fortunately such an instrument was available at Princeton in the mass spectrograph designed by Bleakney [14] and applied by him to the study of hydrogen isotopes. A sample of the gas is bombarded with electrons of controlled potential and the intensities of the resulting positive ions of different masses are measured by the mass spectrograph. A mixture of hydrogen and deuterium may form monatomic, diatomic and triatomic ions of masses varying from one to six. If a third isotope (T) of mass 3 is possible there are further possibilities. The ions up to mass 6 are listed in Table I. By controlling the velocity

TABLE I.

Types of Positive Ions.

Mass.	1.	2.	3.	4.	5.	6.
Ions.....	H^+	$(HH)^+$ D^+	$(HHH)^+$ $(HD)^+$ T^+	$(HHD)^+$ $(DD)^+$ $(HT)^+$	$(HDD)^+$ $(HHT)^+$ $(DT)^+$	$(DDD)^+$ $(TT)^+$

of the electrons producing ionization the formation of monatomic ions as primary products may be suppressed. Their production by secondary collisions will only yield second-order concentrations. The diatomic ions may be distinguished from triatomic ions by studying the intensity of the

[13] Eyring and Topley, *J. Am. Chem. Soc.*, **55**, 5058 (1933); *J. Chem. Phys.* **2**, 217 (1934).

[14] Bleakney, *Phys. Rev.*, **40**, 496 (1932).

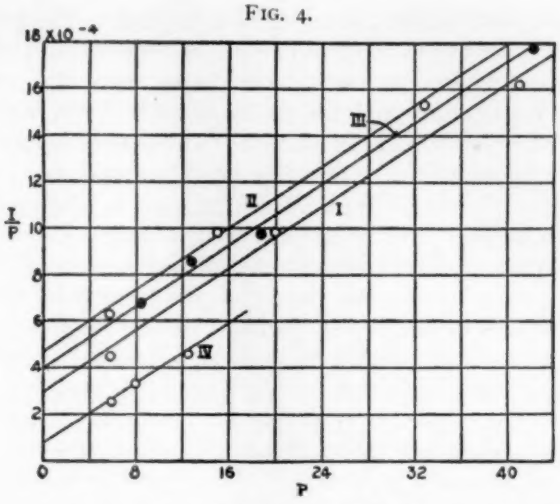
ions as a function of pressure.[15] For example, with ions of mass 3, the intensity of the $(HD)^+$ ions is proportional to the first power of the pressure since they are formed in the primary process. The intensity of $(HHH)^+$ ions is proportional to the second power of the pressure since, as with the monatomic ion T^+ under the chosen conditions of electron bombardment, the production of the triple ions involves a secondary collision. If I is the ionic intensity and p the pressure, a plot of I/p against p should give a straight line whose intercept on the I/p axis represents the intensity due to the diatomic ion $(HD)^+$ in question.

Abundance of the Isotopes in Water.—Using this analytical procedure tests have been made by Bleakney and Gould [16] of the abundance of the deuterium isotope in ordinary water and of the isotope of mass 3 in the pure deuterium oxide prepared in Princeton. Hydrogen-deuterium mixtures liberated from rain water vapor by passage over iron showed a D : H ratio of 1 : 5000. There was a fractional separation of the two gases in the course of the reaction. Initially the D : H ratio was 1 : 6500; the final ratio was 1 : 4300 with the mean concentration already indicated. Figure 4 illustrates the kind of data so obtained. Other reactions have since been shown to give a fractionation of the isotopes as for example, the action of metals on sulphuric acid,[17] sodium on water [18] and the reduction of copper oxide at 200° C. by hydrogen-deuterium mixtures.[19]

By the same mass spectrographic procedure using the gas from electrolysis of the best available deuterium oxide samples Bleakney and Gould [20] have also shown that the ratio of the hydrogen atoms of mass 3 (T) to those of mass 2 (D) cannot be as great as 1 : 50,000 in such heavy water. The passage of the extrapolated curve through the origin in Fig. 5 determines this upper limit of tritium (T) concentration. This means that, even if there is no preferential concentration of tritium

[15] Bleakney, *Phys. Rev.*, **41**, 32 (1932).
[16] Bleakney and Gould, *Phys. Rev.*, **44**, 265 (1933).
[17] A. Farkas and L. Farkas, *Nature*, **133** (1934).
[18] Davis and Johnston, *J. Am. Chem. Soc.*, **56**, 492 (1934).
[19] Hunt, *J. Chem. Phys.*, **2**, 106 (1934).
[20] Bleakney and Gould, *Phys. Rev.*, **45**, 281 (1934).

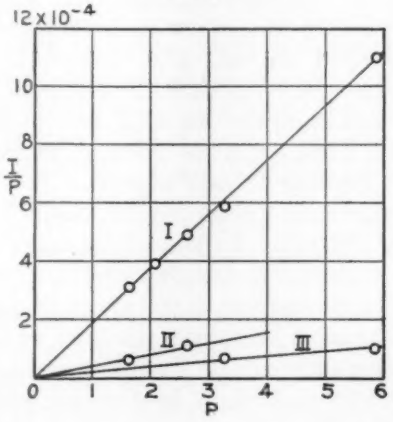
FIG. 4.



The ratio I/p plotted as a function of P.

The intercept on the vertical axis gives the ratio $H^1H^2 : H^1H^1$. Curves I and II represent the first and last fractions decomposed and curve III represents a sample completely decomposed. Curve IV represents a sample of commercial electrolytic hydrogen.

FIG. 5.



in the process of electrolytic production of heavy water, the concentration of T in ordinary water must be less than 1 in 5×10^8. It is thus an exceedingly rare isotope even if the positive evidence in favor of its existence,[21] yielded by the

[21] Latimer and Young, *Phys. Rev.*, **44**, 690 (1933).

Allison magneto-optical method, be accepted. The announcement of its probable formation artificially by deuton bombardment of deuterium-containing substances, recently announced by Lord Rutherford and his co-workers,[22] is not revelant to the abundance estimate in ordinary water here considered.

Cosmic Abundance.—The terrestrial abundance of deuterium is much greater than its abundance in the stars. Examination of the hydrogen lines in the spectra of the outer parts of the sun's chromosphere during a total eclipse led Menzel to the conclusion that the abundance of deuterium in the sun is less than 1 in 500,000 of the total hydrogen.[23] H. N. Russell and Menzel have since extended this observation of what must have been terrestrially, a concentration of the heavier isotope by tremendous preferential loss of the lighter isotope during the process of formation, to the case of other gases, notably neon.[24]

Alternative Methods of Deuterium Concentration.—The mass-spectrograph has also been utilised to follow the concentration of deuterium by other than the electrolytic method. Taylor, Gould and Bleakney[6] studied the desorption of hydrogen gas from charcoal at liquid air temperatures. It was found that a 2.23-fold concentration of HD and a 4.35-fold concentration of D_2 was achieved in the last 50 cc. of gas desorbed from a charcoal initially adsorbing 5 litres of gas. The concentrations achieved are such as can be ascribed to a diffusion process, from a porous body, charcoal, of hydrogen gas adsorbed by van der Waals' forces.

Similar experiments conducted by Gould[25] are concerned with the rates of diffusion of the gases through palladium. It was shown that a gas containing 34 atom per cent. of D, when circulated over a palladium membrane at 270° C. gave an initial diffused gas with 21 atomic per cent. of D, intermediate fractions of 25 and 42 per cent. D and a residue which had not passed through the palladium containing 62.2 per cent. D. The concentration effected is quite small and is explainable on the assumption that the passage through the palladium is a

[22] Rutherford, *Nature*, 133, 481 (1934).
[23] Menzel, *Pub. Act. Soc. Pacific*, 44, 41 (1932).
[24] Menzel and Russell, *Proc. Nat. Acad. Sci.*, 19, 997 (1933).
[25] Gould, Princeton Thesis, 1934.

diffusion of species with masses 1 and 2, and hence that the diffusion is at least atomic and probably ionic, as protons and deutons.

Selwood has examined the practicability of enriching intermediate concentrations of heavy water by fractional crystallisation. He finds that prolonged fractional crystallisation of 90 per cent. D_2O yielded no concentration. Quite evidently the H_2O and D_2O molecules build up equally well into the ice lattice and no difference in concentration between solid and liquid phases occurs.

Mention has already been made of chemical processes of separation. The net result of all such studies is the conclusion that electrolysis is at present the only practical method of separation.

The Hydrogen-Deuterium Equilibrium and its Significance in Surface Catalysis.—From ordinary water, by electrolysis, the major product is molecular H_2 with but minimal amounts of HD molecules. From concentrated deuterium oxide samples the molecules are mainly D_2 molecules. Reaction between H_2 and D_2 molecules yields the mixed HD molecules

$$H_2 + D_2 = 2HD,$$

a reaction between two isotopes of the same element which is, however, obedient to the same laws that govern reactions between different molecular species. The system can proceed to an equilibrium state in which the relative concentrations are expressible by means of the ordinary mass action equation,

$$K = \frac{[HD]^2}{[D_2] \times [H_2]}.$$

Urey and Rittenberg [26] have calculated the equilibrium constant K for various temperatures and have checked their calculations for higher temperatures. To obtain checks at lower temperatures the use of catalysts is necessary. It is obvious that surfaces known for their capacity to induce reaction of hydrogen with other molecular species should be of importance in inducing this interaction of two isotopes of the same element. Mr. Gould, with the analytical assistance of

[26] Urey and Rittenberg, *J. Chem. Phys.*, 1, 137 (1933).

Dr. Bleakney, has examined the capacity of a wide variety of surfaces to induce this change and has obtained results of profound significance in the general problem of surface catalysis.

Surfaces of various glasses, mercury, charcoal, and typical hydrogenation catalysts such as palladium, nickel and chromium oxide have been studied. It was shown that at ordinary temperatures and temperatures below 0° C. the glasses, liquid mercury surfaces and charcoal have no activity in the interconversion of H_2 and D_2 to 2HD. On the contrary, catalytically active preparations of nickel and of charcoal oxide will induce this change *at as low temperatures as liquid air*. The following table shows typical results obtained by Gould on chromium oxide at 0°, −78° and −190° C. starting with mixtures containing mainly H_2 and D_2, except in the final case where the equilibrium was approached from the reverse direction, the initial mixture being rich in HD molecules. The concordance of the observed concentration ratio K' (initially less than 0.03) with the calculated equilibrium constant K is a measure of the efficiency of the surface in the production of equilibrium at the temperature in question.

TABLE II.

Equilibrium Reaction on Chromium Oxide Gel.

Temp. ° C.	Time of Contact in Hours.	Analysis.			K'.	K.
		H_2.	HD.	D_2.		
0....	1	100	95.6	28.7	3.16	3.19
0....	3	100	102	32.2	3.23	3.19
−78....	1	100	100.5	33.8	2.99	2.88
−78....	3	100	99.2	33.9	2.89	2.88
−78....	15	100	99.5	32.9	3.01	2.88
−190....	2	100	41.2	33	0.50	2.2
−190....	26	100	73.1	22.8	2.3	2.2
−190....	25	100	40.2	7.39	2.18*	2.2

* The initial value of K' in this case was 3.57.

The acceleration of reaction thus produced at such low temperatures is not due merely to a concentration of the reactants by van der Waals' adsorption. The surfaces in question are quite specific. Charcoal which shows large van

der Waals' adsorption has no activity. Chromium oxide on the contrary is relatively quite efficient. We are forced therefore to the conclusion that on the surface of chromium oxide, even at liquid air temperatures, there are areas of the surface on which a specific activated adsorption of hydrogen occurs as a prelude to the chemical reaction which occurs. Because equilibrium is established in the gas phase it is evident that the processes of adsorption on and desorption from the areas of the surface in question must occur readily, must therefore have relatively low heats of activation and heats of binding. This conclusion is in marked contrast to that reached in the ordinary studies of activated adsorption in the higher temperature ranges. Here, energies of activation and heats of binding of the order of 20–25 kg. calories were found by Howard [27] for the chromium oxide surfaces he studied. It is quite evident that areas with such characteristics would not permit rapid adsorption and desorption to occur at liquid air temperatures. What these hydrogen-deuterium experiments really demonstrate is that there are areas of low activation energy and low binding energy, capable of producing chemical reaction, which are not revealed by the ordinary processes of study of adsorption phenomena, because their existence is obscured by the preponderant van der Waals', non-specific adsorption occurring at the low temperatures in question. Deuterium is at present the only tool with which the existence of such areas can be made evident.

These results have a real industrial significance. They indicate that the high temperatures, necessary in industrial catalytic syntheses such as those of ammonia and methanol, are required, not for the activation of the hydrogen but for the activation of the molecules with which the hydrogen has to react. If surfaces could be found as active toward these other molecules as present available surfaces are with respect to hydrogen, tremendous improvements in the yields of such products as ammonia and alcohol, under much simpler operating conditions, would be possible. The deuterium experiments indicate the direction which research in industrial catalysis must take.

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[27] Howard, Princeton Thesis, 1933; *Trans. Farad. Soc.*, **30**, 278 (1934).

In agreement with these conclusions actual measurements by Professor R. N. Pease and Mr. Wheeler on the addition of deuterium to ethylene on copper to form $C_2H_4D_2$ and by Taylor and Smith of the reaction of deuterium and carbon monoxide on nickel to form deutero-methane, CD_4, show that there is no marked difference in the rates of these reactions and the corresponding reactions with hydrogen.

The Biochemistry of Heavy Water.—It was obvious that the biological effects of heavy water would quickly receive attention. Lewis [28] found that germination of tobacco seeds was restrained or stopped in heavy water of high concentration. Taylor, Swingle, Eyring and Frost [29] showed that water containing 92 per cent. of the heavy isotope was toxic to tadpoles of the green frog which died within an hour, to the common aquarium fish, the guppy, which died within two hours and to flat worms which died within three hours. Paramœcia and amœba were more resistant; 32 per cent. D_2O solutions were without effect. Considerable discussion of the validity of these results has been raised on the basis that the heavy water may have contained toxic impurities not present in ordinary water. In the case of the guppies they have been repeated with water which has been redistilled twice from alkaline permanganate, the product having a conductivity of 1×10^{-6}, equal to that of excellent distilled water. Moreover, the skeptical have always the burden of explaining why the impurities always gravitate into the 92 per cent. water rather than to the 30 per cent. water in which the animals live excellently, but which water has always been through exactly the same operations both of preparation and purification as has the more concentrated deuterium water.

The study of enzyme reactions in heavy water is revealing some of the reasons for the curious biological effects. Heavy water is not toxic in the way that carbolic acid or potassium cyanide is toxic. It does, however, profoundly modify the speed of biological processes. Yeast ferments α-d-glucose in heavy water at only one-ninth the rate in ordinary water as shown by Professor Pacsu [30] in the following data:

[28] Lewis, *J. Am. Chem. Soc.*, **55**, 3503 (1933).

[29] Taylor, Swingle, Eyring and Frost, *J. Chem. Phys.*, **1**, 751 (1933); *J. Cell. Comp. Physiol.*, **4**, 1 (1933).

[30] Pacsu, *J. Am. Chem. Soc.*, **56**, 245 (1934).

TABLE III.

Alcoholic Fermentation of α-d-glucose.

	H_2O.	60% D_2O.	100% D_2O.
Relative k.....	1	0.6	0.11

Dr. J. Pace has found that the enzyme, catalase, present in the blood stream and whose function it is to destroy hydrogen peroxide, does so at only one-half its normal speed in 85 per cent. D_2O. It is perhaps significant that the most marked effects occur in the range of concentration from 85 to 100 per cent. which was found toxic to the fresh-water organisms.

Professor E. N. Harvey of the Princeton Biology Department has shown [31] that heavy water (85 to 95 per cent. D_2O) does not prevent the luminescence of dried cypridina nor affect the luminescence of a fresh-water bacterium. It does diminish the luminescence of a marine form. It retards the growth of luminous bacteria, sometimes allowing slow growth without luminescence. The oxygen consumption of a salt-water luminous bacterium was reduced 60 per cent. in 86 per cent. heavy water, by 30 per cent. in 63 per cent. water and by 12 per cent. in 36 per cent. heavy water. The respiration of a fresh-water bacterium was less markedly affected. The curves relating reduction of respiration and concentration of heavy water would indicate zero respiration in 100 per cent. heavy water.

Heavy water was shown to kill a number of protozoa and rotifers but does not kill bacteria or injure Euglena irreversibly. It affects only slowly the protoplasmic rotation of Elodea cells and it has been shown to penetrate into such Elodea cells. In general Preofessor Harvey concludes that the action of heavy water may be likened to that of a generally unfavorable environment, leading to progressive changes in the cell. It would seem that the changes observed are the result of the differential effects on the rate of biochemical reactions already mentioned in connection with the enzyme reactions.

[31] Harvey and G. W. Taylor, *J. Cell. Comp. Physiol.*, 4, 357 (1934); Harvey, *Biol. Bull.*, 66, April (1934).

Deuterium-Containing Compounds.—The large number of known chemical compounds containing hydrogen at once suggests an overwhelming program for the production of compounds in which deuterium has replaced hydrogen. Such a program, however, if judiciously conducted, will aim rather at producing new materials with which important problems of physico-chemical science may be tested. Two examples from the Princeton researches will be cited, the one to illustrate a problem in the organic chemistry of optically active compounds, the other to illustrate problems in preparative inorganic chemistry, the theory of liquids and modern photochemical problems.

Professor E. Pacsu [32] has studied the mutarotation of α-d-glucose in water containing 60 and 100 per cent. of the hydrogen as deuterium. His experimental results are exhibited in the following table.

TABLE IV

Mutarotation of α-d-Glucose.

H_2O.	60% D_2O.	100% D_2O.	Observer.
$k_1 + k_2$ = 0.00546(18°) 0.00635(20°) 0.00225(10°)	0.00290(18°)	0.00221(20°)	Pacsu Hudson and Dale

It is quite evident that the velocity of mutarotation in pure D_2O at 20° C. is identical with that found in ordinary distilled water at 10° C. In this case therefore the substitution of deuterium for hydrogen has an effect on the velocity equivalent to a fall of 10° in temperature. This result is in best accord with the view that the mutarotation of sugars is due to the wandering of a mobile hydrogen atom, the decrease in mobility by substitution of D for H being in accord with the predicted influence of zero point energy in such cases.[4] Pacsu concludes that, as the lower velocity in heavy water is observed even in the first minutes of the change, the structural change in the molecule due to displacement of hydrogen by deuterium occurs immediately. Pacsu also sees in his results confirmation of the view that mutarotation is due to

[32] Pacsu, *J. Am. Chem. Soc.*, **55**, 5056 (1933).

what is termed oxy-cyclo desmotropy, that is, to a change in which the double bond of a $\diagdown\!\!\diagup C = O$ group is replaced by a ring formed by the migration of a hydrogen atom. Also, he finds that the specific rotation of the equilibrium solution of the glucose in heavy water is identical with that in distilled water. Evidently the replacement of the hydrogen atom by a deuterium atom in glucose does not change the value of the specific rotation for the wave-length of sodium light. This is an observation of importance in the problem of the possible production of optically active molecules containing atoms which are asymmetric due to binding with two isotopic forms of one element.

Taylor and Jungers [33] have shown that the interaction of deuterium oxide vapor with magnesium nitride may be employed to produce the deuterium containing ammonia ND_3. The investigation of this method of preparation revealed circumstances in connection with the preparation of magnesium nitride which had hitherto escaped observation. It was found that the product formed by interaction of ordinary ammonia with metallic magnesium at dull red heat, followed by displacement of the excess ammonia through evacuation at room temperatures is not actually magnesium nitride, Mg_3N_2, but must contain a considerable volume of bound hydrogen. For, on treatment of such material with pure D_2O the product, instead of being pure ND_3, was a mixture of the deutero-ammonias ND_3, ND_2H and NDH_2 in which only 70 per cent. of the hydrogen was the heavy isotope. A second sample of the magnesium preparation similarly obtained but which was subjected to prolonged evacuation for 40 hours at 400° C. still contained enough bound hydrogen to yield a deutero-ammonia in which 10 per cent. of the hydrogen was light isotope. Actually, the pure deutero-ammonia ND_3 was prepared by the interaction of deuterium oxide with a nitride sample prepared by the interaction of magnesium metal, not with ammonia, but with purified nitrogen. It is therefore obvious that the formation of nitride from magnesium and ammonia must occur via quite stable amido- or imido-inter-

[33] Taylor and Jungers, *J. Am. Chem. Soc.*, **55**, 5057 (1933).

mediate stages. In this respect it is similar to tungsten powder.[34]

The following Table V shows that similar deviations in the physical properties of the ammonias obtain as in the case of ordinary and heavy water.

TABLE V.

Physical and Photochemical Properties of Deuteroammonias.

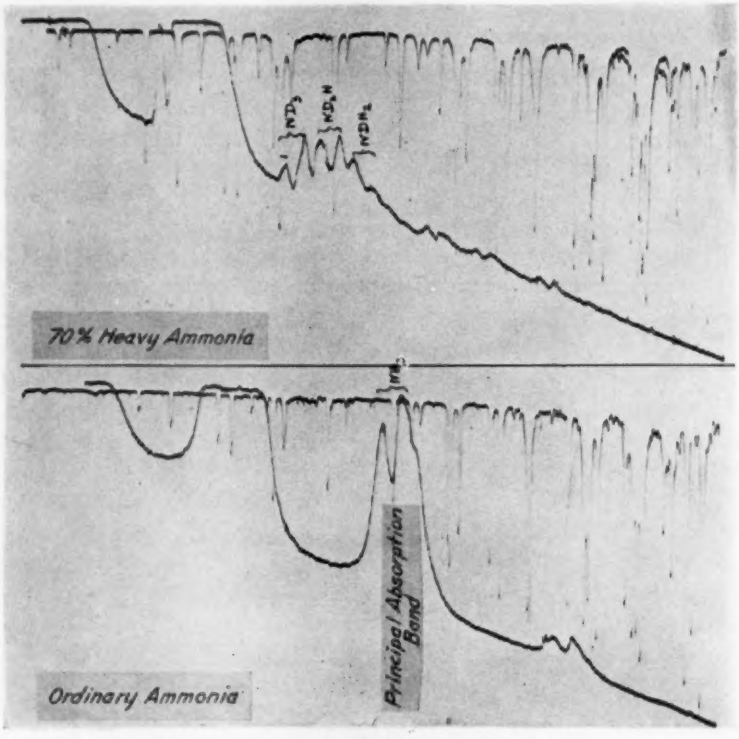
Percentage of Heavy Hydrogen.....	0.	68.	90.	>99.
Freezing point, ° Abs.....	195.2	197.9	198.6	199
Boiling point, ° Abs.....	239.75	241.7	242.1	242.3
Hg sensitized decomposition $\Delta p/30$ mins.....	11.3	2.55	1.55	1.11

Thus, the freezing point of ND_3 is 3.8 degrees higher than that of NH_3, the boiling points differ by 2.5°. The intermediate concentrations give intermediate boiling and freezing points. In the ammonias containing 68 and 90 per cent. D, there are present the three deuteroammonias NH_2D, NHD_2 and ND_3 in proportions determined by the deuterium concentration. This is at once revealed by the absorption spectra of the several preparations in the ultraviolet region. In Figs. 6 and 7 are shown spectrophotometric curves of ordinary ammonia and deuteroammonia (70 per cent. D) in the one case and of 70 and 90 per cent. D ammonias in the other. In Fig. 6 the displacement of the principal absorption band to shorter wave-lengths in the case of the heavy ammonia is evident. The complexity of this band as compared with that of the ordinary ammonia is also apparent. Dr. Benedict has examined these photometric curves and has found that the complexity is due to the presence of all three deuteroammonia molecules, the displacement to shorter wave-lengths being greatest with the ND_3 molecule and least with the NDH_2 molecule. From the deuterium content of the sample it can be computed that, in a 70 per cent. D sample, there should be present 34 per cent. ND_3, 44 per cent. ND_2H, 19 per cent. NDH_2 and 3 per cent. NH_3. The spectrophotometric curve of the bands is in fair agreement with this calculation, so that we have here a useful semiquantitative physical method

[34] Frankenburger and Hodler, *Trans. Farad. Soc.*, **28**, 229 (1932).

of exhibiting the effect of isotope substitution in the molecule. In the 90 per cent. D sample, the ND_2H concentration should, by calculation, have fallen to approximately 25 per cent. and that of NDH_2 to 3 per cent. The corresponding curves in Fig. 7 show that the doublets corresponding to ND_2H and

FIG. 6.

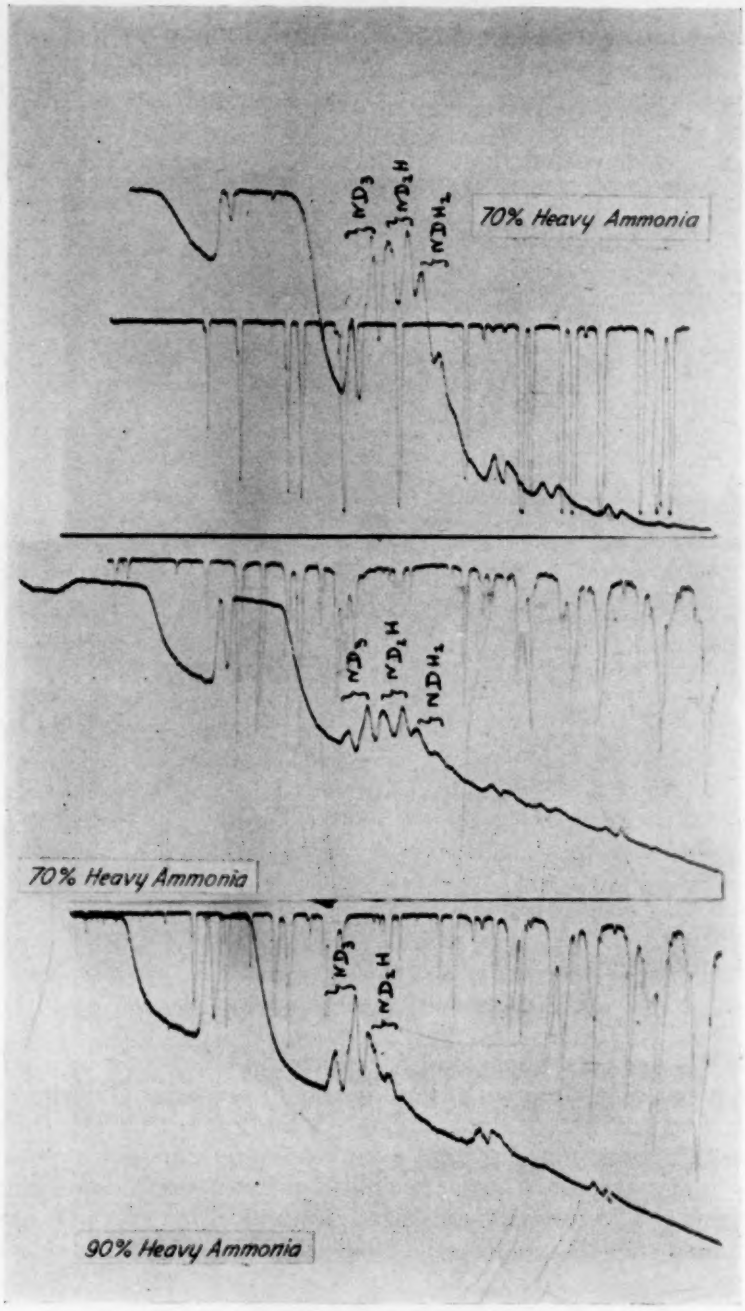


Absorption spectra of ammonia and deuteroammonias.

NDH_2 are very considerably reduced in the 90 per cent. D curve as compared with those in the 70 per cent. D curve.

The last line of data in Table V indicates that the three deutero-ammonias have all a higher degree of stability than ordinary ammonia towards collisions from excited mercury atoms. The mercury-sensitized decomposition of these ammonias was chosen for study by Jungers and Taylor rather

FIG. 7.



Absorption spectra of 70 per cent. and 90 per cent. deuteroammonias.

than the simple photo-decomposition because, as the absorption spectra reveal, the absorptions of the several ammonias are so different that it would be more difficult to secure strictly comparable conditions of experiment. With excited mercury, collisions of ammonia were known principally to convert the excited mercury atoms from the excited 2^3P_1 to the metastable 2^3P_0 state. Experiments by Dr. M. G. Evans have shown that the ND_3 molecules produce this same transition but only with approximately one-third of the collision efficiency. As the experimental results show, the rate of photosensitized decomposition is 10 times as great with ammonia as with ND_3. In both cases it is found that the hydrogen and deuterium produced by the decomposition have a most pronounced inhibiting action on the decomposition process. As was first pointed out by Mitchell and Dickinson [35] this is to be attributed to the great efficiency of hydrogen in quenching mercury fluorescence by taking the excitation energy of the 2^3P_1 mercury atom for dissociation of the hydrogen molecule to atoms, returning the mercury atom to its normal 1S state. Quenching experiments by Evans show that hydrogen and deuterium are equally efficient in quenching the fluorescence of excited mercury. Even correcting, however, for the inhibiting actions of hydrogen and deuterium it is found that the *initial* photosensitized decomposition of ND_3 is ten times smaller than that of NH_3. Were the collisions which drive the mercury to the metastable state alone responsible for the activation of the ammonias, these could, as has been said, only account for one-third of this 10-fold stability difference. It would be necessary to account for the remainder by means of the difference in zero point energies of the two ammonia molecules. There is some doubt as to whether the transition of the mercury from the excited to the metastable state yields enough energy to produce the decomposition of the ammonias which finally results. Other less frequent transitions may be involved. Sufficient facts however, have already been accumulated to exhibit the differences in photochemical stability associated with isotope changes.

Rollefson of California has shown [36] that the photo-

[35] Mitchell and Dickinson, *J. Am. Chem. Soc.*, **49**, 1478 (1927).

[36] Rollefson, *J. Chem. Phys.*, **2**, 144 (1934).

chemical reaction of deuterium and chlorine at ordinary temperatures proceeds more slowly than the corresponding reaction involving hydrogen. In agreement with Eyring's calculations on zero point energy,[37] this result is undoubtedly due to the slower reaction of chlorine atoms with deuterium,

$$Cl + D_2 = DCl + D$$

than with hydrogen

$$Cl + H_2 = HCl + H$$

due to the smaller zero point energy of the deuterium. The ratio of velocities was 1 to 10. This same zero point energy effect may be used to decide a doubtful case of photochemical mechanism. In the mercury photosensitized combination of hydrogen and oxygen to produce hydrogen peroxide it was suggested by Taylor [38] that the mechanism might be

$$Hg' + H_2 = Hg + 2H \qquad (1)$$

$$H + O_2 \rightarrow HO_2 \qquad (2)$$

and

$$HO_2 + H_2 \rightarrow H_2O_2 + H. \qquad (3)$$

On the basis of zero point energy, and in agreement with Rollefson's result above, reaction (3) might be expected to proceed more slowly if deuterium were the reactant instead of hydrogen. Now Evans finds that there is no difference in reaction rate when D_2 is substituted for H_2 in this reaction. This result makes reaction (3) quite doubtful and suggests that an alternative mechanism proposed by Bates [39]

$$HO_2 + HO_2 = H_2O_2 + O_2 \qquad (3a)$$

may be the reaction producing peroxide. In this reaction (3*a*) no marked difference between hydrogen and deuterium would be expected.

The Physics of Heavy Hydrogen.—The outstanding use to which deuterium has been put in research in the physical laboratory is as a bombarding ion in atomic transmutation.

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[37] Eyring and Sherman, *J. Chem. Phys.*, 1, 345 (1933).
[38] Taylor, *Trans. Farad. Soc.*, 21, 560 (1925).
[39] Bates, *J. Chem. Phys.*, 1, 457 (1933).

Immediately after the isolation of deuterium oxide, Lewis, Lawrence and Livingstone [40] showed that deutons, D^+, the nuclei of deuterium atoms, were markedly more efficient than protons in producing atomic disintegrations, notably of lithium. It was shown that transmutations involving higher yields of atomic energy were obtained than in the case of protons. Lord Rutherford, Oliphant and Kinsey [41] and Dee and Walton [42] investigated this more thoroughly and the paths of the disintegrating fragments were revealed by the latter workers as cloud tracks in a Wilson expansion chamber, which could then be recorded photographically. The several methods of study lead to two alternative distintegration processes with Li^6 and Li^7 respectively. They may be represented by the nuclear equations

$$Li^6 + D^2 = 2He^4$$

and

$$Li^7 + D^2 = 2He^4 + n'$$

where n' represents the neutron. Rutherford estimates that the maximum energy of the ejected neutron may be as great as 15 million volts. Lauritsen and Crane [43] have found high yields of neutrons from the bombardment of beryllium with deutons. The most recent of the reports concerning such transmutations [44] is that deuton bombardment of deuterium-containing compounds may give rise to the hydrogen isotope of mass 3, concerning whose small concentration in natural sources we have already presented the experimental evidence. Lord Rutherford assumes that a deuton occasionally strikes a deuton sufficiently closely to form a helium nucleus of mass 4 but containing a large excess of energy. The new nucleus is in consequence explosive and breaks up into two parts, one a fast proton, the other the nucleus of the hydrogen isotope of mass 3. Calculation shows that the range of the new nucleus should be about 1.7 cm. in air, a range agreeing closely with that actually observed, 1.6 cm. The change would be repre-

[40] Lewis, Lawrence and Livingstone, *Phys. Rev.*, **43**, 55 (1933).
[41] Rutherford, Oliphant and Kinsey, *Proc. Roy. Soc.*, **141A**, 722 (1933).
[42] Dee and Walton, *Proc. Roy. Soc.*, **141A**, 733 (1933).
[43] Lauritsen and Crane, *Phys. Rev.*, **45**, 493 (1934).
[44] Rutherford, *Nature*, **133**, 481 (1934).

sentable by the equation

$$D_1^2 + D_1^2 \rightarrow He_2^4 \rightarrow T_1^3 + H_1^1.$$

In this nuclear equation the superscript denotes the mass and the subscript the charge. From the energy relations involved and the masses, the mass of the new isotope would be 3.0151. There is also evidence of a neutron-emitting process for which the equation would be

$$D_1^2 + D_1^2 \rightarrow He_2^4 \rightarrow He_2^3 + n_0^1,$$

which therefore would produce a helium isotope of mass 3. Lord Rutherford finds evidence for this isotope with a mass of 3.0165 when lithium of mass 6 is bombarded with protons.

The quite recent experiments on artificially produced radioactivity initiated by Curie and Joliot [45] using alpha particles are also being supplemented by similar studies using high speed deutons. Lauritsen and Crane [43] observe delayed decomposition of some of the products from the bombardment of a target of B_2O_3 with deutons. They suppose that the reaction is

$$B^{10} + D^2 \rightarrow C^{11} + n',$$

the C^{11} being the radioactive product. This is, however, assumed to be subordinate (1/40) to the main nuclear reaction

$$B^{11} + D^2 = C^{12} + n' + \gamma \text{ radiation.}$$

Protons produced delayed activity also but only to the extent of 10 to 20 per cent. of that by deutons under similar circumstances.

In all these calculations of atomic transmutations the masses of the various nuclei and of the neutron are of first importance owing to the Einstein relation between energy produced and mass annihilated. Ordinarily a value of approximately 1.0067 is assumed for the mass of the neutron, which gives for the mass of 1 proton and 1 neutron a value 1.0078 + 1.0067 = 2.0145. A deuton of mass 2.0131 produced from these would give up the equivalent of this mass difference as energy and the deuton might be expected to be

[45] Curie and Joliot, *Compt. Rend.*, **198**, 254 (1934).

stable. Some data from Lawrence in California suggested, however, a much lower value of 1.0006. If this were the correct value one might expect the deuterium nucleus to be quite unstable, since it has a mass of 2.0131 as compared with the masses of a proton and a neutron which, on the value of Lawrence, would be 1.0078 + 1.0006 = 2.0084. The difference would have to be present as stored up energy. Experiments in Princeton by Professor R. Ladenburg [46] on the radioactivity of heavy water showed no evidence of any such instability as might have been expected on the basis of the lower value for the neutron, but on the contrary a stability which may be stated in terms of a "half life" exceeding 10^{15} years. These experiments make the low value for the mass of the neutron more improbable, and Lewis, Livingston, Henderson and Lawrence [47] now believe that it is possible that alternative explanations may be found for the phenomena which originally led to the hypothesis of the instability of the deuton.

In spectroscopy the isotope effect has been of great service in elucidating a number of spectroscopic problems. The discovery of the hydrogen isotope makes possible a much more detailed study of the effect of a change in mass on the energy levels of molecules, because owing to the greater relative change in mass between H and D the isotope effect is much bigger than for all other cases studied previously. Refinements of isotope theory must now be considered and may be examined experimentally. Urey, Brickwedde and Murphy [48] defined the existence of deuterium by examining the weak Balmer lines of D accompanying the much stronger lines of H. Numerous preparations have now been examined in which the D Balmer lines are the strong lines and the H lines the weak.

Of diatomic molecules containing deuterium there have been examined DCl, HD, D_2, OD and ND. Hardy, Barker and Dennison [49] examined 19 lines of the DCl^{35} molecule and 17 of the DCl^{37} molecule. From their measurements they

[46] Ladenburg, *Phys. Rev.*, **45**, 224 (1934).
[47] Lewis, Livingston, Henderson and Lawrence, *Phys. Rev.*, **45**, 497 (1934).
[48] Urey, Brickwedde and Murphy, *Phys. Rev.*, **39**, 164, 864 (1931).
[49] Hardy, Barker and Dennison, *Phys. Rev.*, **42**, 279 (1932).

deduced a mass of 2.0137 in close agreement with the mass spectrographic value of Bainbridge, 2.0131. Miss Ashley and G. N. Lewis [50] obtained 21 foot grating spectrograms of a gas containing H_2, HD and D_2. More recent measurements by Dieke and Blue [51] have yielded a detailed analysis of the Fulcher bands of HD and D_2 molecules and have given new information about the H_2 levels. The D_2 molecule shows the alternating intensity due to ortho and para forms with the intensities in the ratio 1 : 2 due to the spin quantum number of the deuterium nucleus of 1 as opposed to that of the H nucleus of 1/2. The ortho form is the stable form at low temperatures. Johnston and Dawson [52] have identified and analysed large portions of the 0,0; 0,1; 1,0; and 2,0 bands of the OD molecule. A corresponding analysis of several ND bands is due to Dieke and Blue.[53]

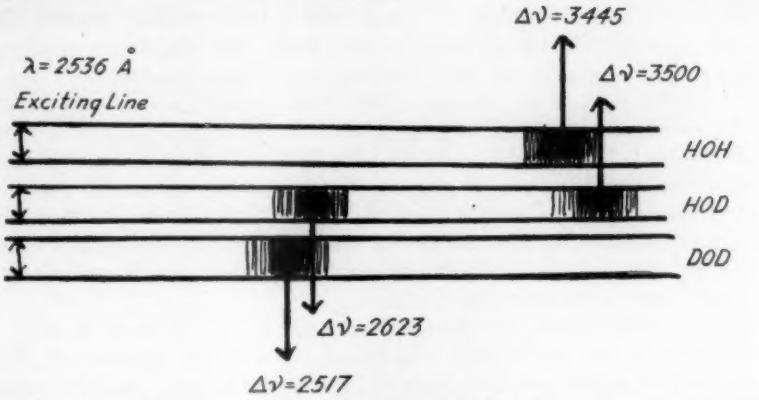
In polyatomic systems the principal molecules that have been examined are the substituted acetylenes CH ≡ CD and CD ≡ CD and the deuteroammonias. The ultraviolet absorptions of the latter have already been discussed. Silverman and Sanderson [54] have examined the near infra-red absorption of ND_3 while the pure rotation spectrum of the same molecule as well as that of NH_3 have been preliminarily examined by Barnes, Benedict and Lewis.[55]

R. W. Wood [56] has examined the Raman spectrum of liquid water with results that are expressed diagrammatically in the accompanying Fig. 8. Experiments on the vapor are promised.

The foregoing pages make no attempt at an exhaustive treatment of the literature of heavy hydrogen. In so many fields, the existence of an isotope of an element, in quantity, in a state of reasonable purity, is an asset of very extensive utility. It is not surprising, therefore, that in many labora-

[50] Lewis and Ashley, *Phys. Rev.*, **43**, 770, 837 (1933).
[51] Dieke and Blue, A. C. S., St. Petersburg Fla. meeting, March, 1934.
[52] Johnston and Dawson, *Phys. Rev.*, **44**, 1031 (1933).
[53] Dieke and Blue, *Phys. Rev.*, **45**, 395 (1934).
[54] Silverman and Sanderson, *Phys. Rev.*, **44**, 1032 (1933).
[55] Barnes, Benedict and Lewis, *Phys. Rev.*, **45**, 347 (1934).
[56] R. W. Wood, *Phys. Rev.*, **45**, 392 (1934).

FIG. 8.



Diagrammatic representation of the data of R. W. Wood on the Raman spectra of the molecules HOH, HOD, and DOD.

tories the new isotope is being vigorously produced and applied. The foregoing pages deal, in the main, with those aspects of the problem which were of most concern to a large group of research men in Princeton. They suggest, it is believed, a much more extensive program of research now actively in progress.

Misbranded Foods and Drugs.—The Federal Food and Drug Administration is seldom if ever idle. Its activities during the month of March is good evidence that many manufacturers continue to be lax regarding the correct labelling of their products.

In the field of drugs, fines have been imposed for the interstate shipment of citrate of magnesia incorrectly labelled U. S. P. and also short in volume. Dr. H. Haehle's "Volcanic Earth" was found not to have curative properties in respect to certain diseases and disorders. "Cascara Cold Breakers" contained less than the stated quantity of acetanilid. A consignment of "Sweet Spirits of Nitre" was not up to standard and "Pa-Poose Root Beer Extract" could never live up to its claims.

The fact that foods can spoil so easily keeps the food inspectors continually on the watch. A month's roster of confiscated shipments includes tomato paste that is decomposed; tomato puree found to be moldy; celery and apples bearing excessive arsenic and lead spray residues; frozen strawberries and frozen eggs partially decomposed; butter which is below standard; "Real Fruit Juice" which was not real, having been diluted with sugar and water with citric acid and sodium benzoate added; "cane syrup" which tasted like and was corn syrup; and lastly, a consignment of black pepper which was not so hot as the label indicated.

C.

THE BLAIR-LEIGHTON EQUATION APPLIED TO THE PRODUCTION OF THE LATENT PHOTOGRAPHIC IMAGE BY HYDROGEN PEROXIDE.

BY

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University of Colorado.

In studies of the rate of formation of the latent photographic image, Blair and Leighton [1] found that the experimental data were adequately represented by the equation:

$$dx/dt = K_1 x^{\frac{1}{2}}(b - x) - K_2 x,$$

where x is the developable density existing at time, t, K_1 and K_2 are the forward and reverse reaction velocity constants and b is the ideal maximum developable density i.e. the density which would result if all the silver halide in the emulsion were developed. Originally evolved from studies of the production of the latent image by heat,[2] this equation has since been shown to be valid in the case of exposure to light [1] and to X-rays.[3] It also furnishes a reasonable explanation of the various aspects of the Herschel effect, photoregression and other characteristics of the latent image.

The application of the equation to the rate of formation of the latent image by other agents is now under investigation. Sheppard and Wightman [4] have published data upon the formation of the latent image by hydrogen peroxide. In the present paper, an attempt is made to show that the equation fits their data and additional experimental observations of interest are presented.

Two lots (of different emulsion number) of Eastman 33 plates, which are similar to those used by Sheppard and Wightman, were treated as described in their paper. That

[1] Blair and Leighton, *J. Phys. Chem.*, **36**, 1649 (1932).

[2] Blair and Leighton, forthcoming paper.

[3] Lester and Blair, *J. Phys. Chem.*, **37**, 17 (1933).

[4] Sheppard and Wightman, *J. Frank. Inst.*, **195**, 337 (1923).

is they were immersed for measured periods in 0.3 M hydrogen peroxide, then washed for thirty minutes in running water and developed for five minutes in a pyro-soda developer, all at the same temperature, 20° C. They were rinsed briefly in 1 per cent. acetic acid, fixed, washed and dried in the usual manner.

The densities produced were measured with the densitometer described by Blair and Leighton.[1] The accompanying data table and the points of Fig. 2 show averages of the densities produced by stated immersions in hydrogen peroxide. In Fig. 2, the time axis is arbitrarily condensed after forty seconds, as indicated. Aside from two empirical curves drawn for the fragmentary data obtained from Process film and Speedway plates, the smooth curves are all drawn through points computed on the basis of the Blair-Leighton equation.

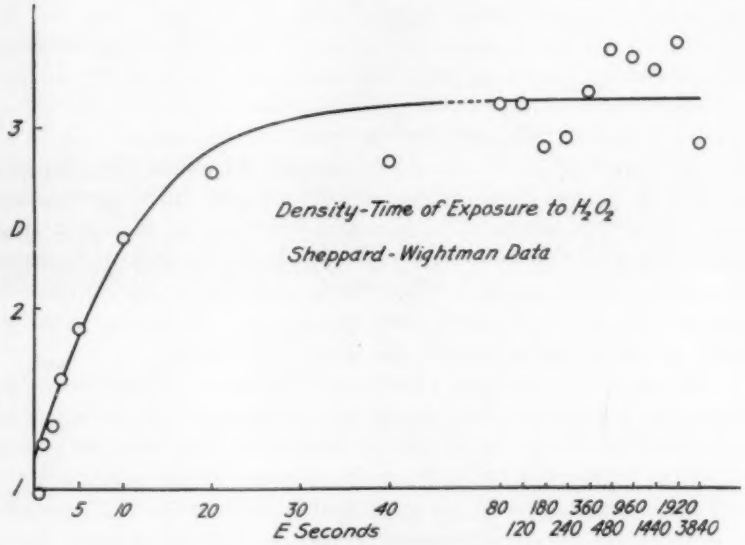
In order to find a comparatively accurate value for the ideal maximum developable density, plates were exposed to direct sunlight for the optimum time and finished in the usual manner. The density obtained in this manner closely approximates the ideal maximum developable density determined by other means.

The densities which Sheppard and Wightman obtained by five minute development after varying exposures to hydrogen peroxide are shown by the points on Fig. 1. No attempt was made to analyze the data which they obtained after less complete development. The time axis is condensed as indicated, for times greater than forty seconds. From these points, the slope at time 7.5 seconds and density 2.15 is found to be 0.10. Neglecting experimental variations, the average maximum density obtained is 3.15, at which value the slope is 0. In the absence of a directly accessible experimental value, 5.82 * is assumed as the ideal maximum

* This quantity is the fourth member of a proportion between the average maximum density, 2.37, obtained by immersion in hydrogen peroxide solution, of the two emulsions of Eastman 33 plates, the corresponding quantity, 3.15, from the data of Sheppard and Wightman and the experimentally determined average ideal maximum developable density, 4.38, of the two emulsions studied in this laboratory. In view of the lack of correlation between sensitivity to light and to hydrogen peroxide pointed out below, this value is doubtless somewhat in error. However, small variations in the value of the ideal maximum developable density do not greatly affect the computed curve and 5.82 is undoubtedly of the right order of magnitude.

developable density, b, for the emulsion they used. Substituting these values in the differential equation given above, the forward and reverse reaction velocity constants are found to be 0.0465 and 0.0699, respectively. When another substitution is made in an integrated form of the equation, using the values just mentioned for K_1 and K_2, together with $x = 2.15$ at $t = 7.5$, the constant of integration, C, is found to be 6.35. The smooth curve in Fig. 1 shows the relations

FIG. 1.



between density and time of exposure computed from the last equation, using the constants previously mentioned.

In the paper already referred to, Sheppard and Wightman present a smooth curve drawn through these same experimental points. The maximum deviation from their curve is greater than from the curve based upon theoretical postulates which is presented here. The more or less random scattering of points for longer exposures quite possibly indicates experimental error. This interpretation is supported by the absence of deviations of similar kind and magnitude either in single observations or in the composite values for the experimental

data from this laboratory. Other preliminary data, not included in this paper because of early difficulty in accurately controlling washing and development temperatures, also agree among themselves in indicating lack of appreciable or consistent variations from the horizontal portion of the curve. No evidence of solarization by hydrogen peroxide is found in these data although other Eastman 33 plates of different emulsion number show evidence of it.

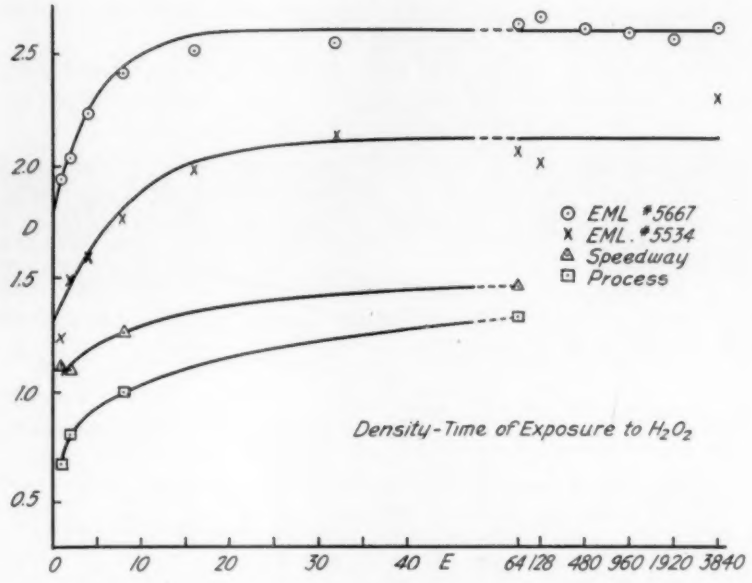
After very short immersions in hydrogen peroxide solution, the developable densities are, in general, less than those predicted by the equation. Such deviations are to be expected. After immersion for the given time, a diminishing amount of hydrogen peroxide is present in the emulsion during the ensuing washing period. This tends to produce a developable density. Extrapolation from experimental results to an exposure of zero duration would indicate the density produced in the wash water upon a plate fully permeated with hydrogen peroxide. Equally obvious is the fact that penetration of the hydrogen peroxide into the gelatine cannot be quite instantaneous. This, then, would account for experimental densities produced by exposures up to one, two or three seconds falling below the predicted values.

Preliminary work in which two different emulsions (No. 5534 and No. 5667) of Eastman 33 plates were used indicated considerable difference in the behavior of the two emulsions. Identical exposures to light, with a sector wheel sensitometer and otherwise, showed no appreciable difference in the sensitivity of the two emulsions. Careful determinations, however, leave no doubt of a large difference in sensitivity to hydrogen peroxide as shown in the data table and the two upper curves of Fig. 2. No explanation of this difference in sensitivity to hydrogen peroxide is attempted in the absence of details of the manufacture of the two batches of emulsion.

Next, the converse question, whether great differences in light sensitivity have any general connection with behavior toward hydrogen peroxide, was investigated. Process film and Speedway plates were used as examples of extremes of light sensitivity. The Speedway plate used was found to be slightly more sensitive to hydrogen peroxide than the Process film but the densities produced upon these unlike emulsions

by simultaneous immersions for equal periods differ less than those upon the two different emulsions of Eastman 33 plates. The Speedway emulsion shows great irregularity of density and clumping of the blackened grains. There is also comparatively heavy fog on the portion not immersed in the hydrogen peroxide solution but more or less contaminated by the vapor and the very dilute hydrogen peroxide in the wash water. It is also interesting to note that the densities

FIG. 2.



attained on both the Speedway and Process emulsions are, in general, less than on either emulsion of Eastman 33 plates. The above results tend to confirm the conclusion, implied in the preceding paragraph, that sensitivity to hydrogen peroxide is not related in any simple way to light sensitivity.

That the rate of formation of the latent image by hydrogen peroxide follows an equation developed for the formation of the latent image by heat and found valid for light supports the hypothesis that these processes are fundamentally identical. This viewpoint is further substantiated by an observa-

tion of the writers that the latent image formed by hydrogen peroxide, like those formed by light and heat, is subject to physical development.

SUMMARY.

1. Data published by Sheppard and Wightman are interpreted as supporting the Blair-Leighton equation.

2. Data presented here also indicate that the rate of formation of the latent image by hydrogen peroxide follows the same equation.

3. Variations of the data of Sheppard and Wightman from the horizontal portion of the theoretically computed curve are not confirmed by data from this laboratory.

4. Evidence is presented that there is no direct correlation between the behavior of emulsions upon exposure to light and to hydrogen peroxide for (*a*) emulsions identical in their reaction to light differ widely in their sensitivity to hydrogen peroxide and (*b*) emulsions differing widely in their light sensitivity show little difference in their behavior with hydrogen peroxide.

5. Evidence is presented which tends to show that the process of production of the latent image by different agents is fundamentally identical.

DATA TABLE.

Time of Immersion in 0.3 M H_2O_2, in Sec.	Density.			
	Eastman 33 Plates.		Speedway Plate.	Process Film.
	Eml. No. 5667	Eml. No. 5534.		
1	1.94	1.23	1.108	0.668
2	2.03	1.49	1.087	0.800
4	2.23	1.59		
8	2.41	1.76	1.258	0.990
16	2.51	1.98		
32	2.55	2.14		
64	2.64	2.07	1.47	1.33
128	2.67	2.02		
480	2.62			
960	2.60			
1920	2.57			
3840	2.62	2.30		

LAW OF THE GEOMETRIC MEAN AS A SENSITOMETRIC FUNCTION.

BY

C. C. LIENAU,

College of Engineering, West Virginia University.

The law of the geometric mean, which is useful in describing the sensitivity of the eye,[1] also finds analogous application as a sensitometric function for photographic emulsions. The density-exposure relation may thus be regarded as an integral probability given by the generalized law of the geometric mean, provided reversal be ignored. This law requires the density $D(E)$ to increase from zero to a maximum, D_m as E, the exposure, increases indefinitely from E_0. We then have

$$\frac{D(E)}{D_m} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{S} \text{exp.} \left(-\tfrac{1}{2}S^2\right) dS, \tag{1}$$

where

$$S = S_0 \log_e \frac{E - E_0}{E_g - E_0}, \tag{2}$$

$$\left.\begin{array}{r} E > E_0 \\ E_g > E_0 \\ S_0 > 0 \end{array}\right\}. \tag{2a}$$

The parameters are given by

when
$$\left.\begin{array}{l} E = E_g \\ D = D_m/2 \end{array}\right\}, \tag{3}$$

when
$$\left.\begin{array}{l} E = E_0 \\ D = 0 \end{array}\right\} \tag{4}$$

and S_0 by

$$\left.\begin{aligned} E &= E_1 \\ S &= 1 \\ \frac{D_1}{D_m} &= 0.84131\cdots \end{aligned}\right\}. \tag{5}$$

When

$$|E - E_g| \to 0;$$

the integral gives

$$\left.\begin{aligned} D &= \gamma' \log_e (E - E_0) + \text{const.} \\ \gamma' &= \frac{D_m}{\sqrt{2\pi}} S_0 \end{aligned}\right\}. \tag{6}$$

where

On logarithmic probability paper, when $E_0 = 0$ the plot of equation (1) yields a straight line; when $E_0 > 0$, E_0 is a vertical asymptote (1) being convex upward with respect to the E axis; when $E_0 < 0$ the curvature is opposite. E_0 may be estimated by rectifying the curve. The scale of S is given by uniform distances along the probability axis with the null point at $D = D_m/2$. See Fig. 1.

Taking as example some Hurter-Driffield data [2] and assuming that D_m is given approximately by the maximum density before reversal:

$$\begin{aligned} D_m &= 3.52, \\ E_0 &= 0, \\ E_g &= 100, \\ S_0 &= 0.426, \\ \gamma' &= 0.596. \end{aligned} \tag{7}$$

The results are shown graphically in two way, Fig. 2 representing equations (1) and (6) on rectangular coordinates.

INTERPRETATION.

It will be recalled that size-frequency distributions of sensitive particles often follow equation (1) and that sensitivity is, partly at least, a function of size.[3] It will also be observed that S is identical in form with the entropy function, involving at the same time a parameter E_0 which can char-

FIG. 1.

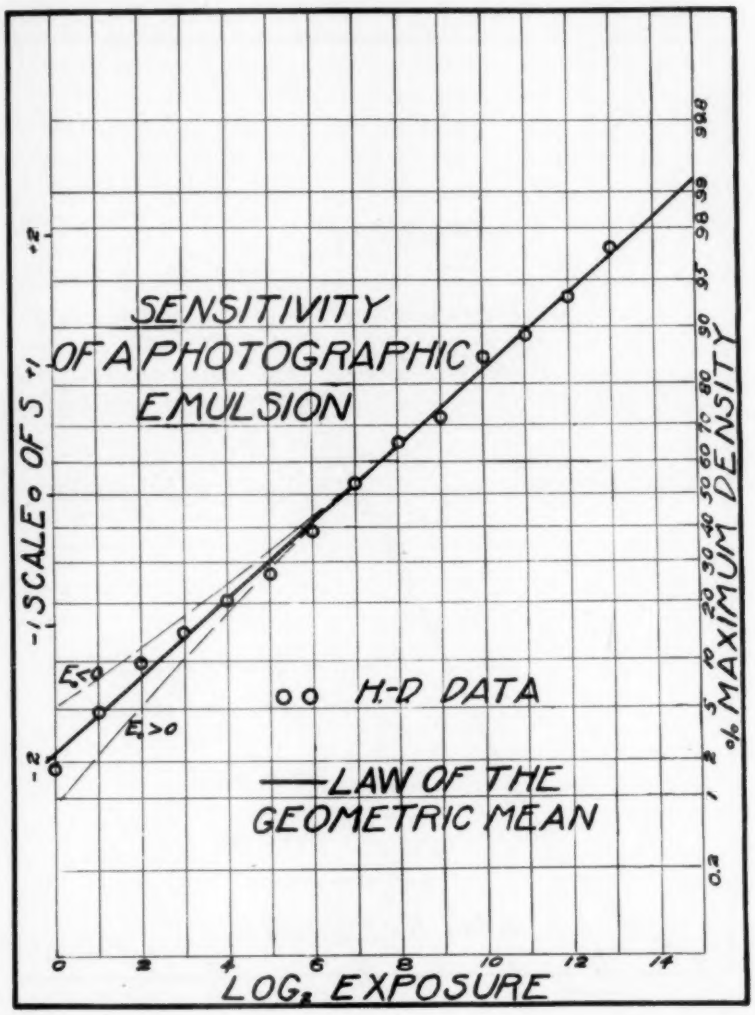
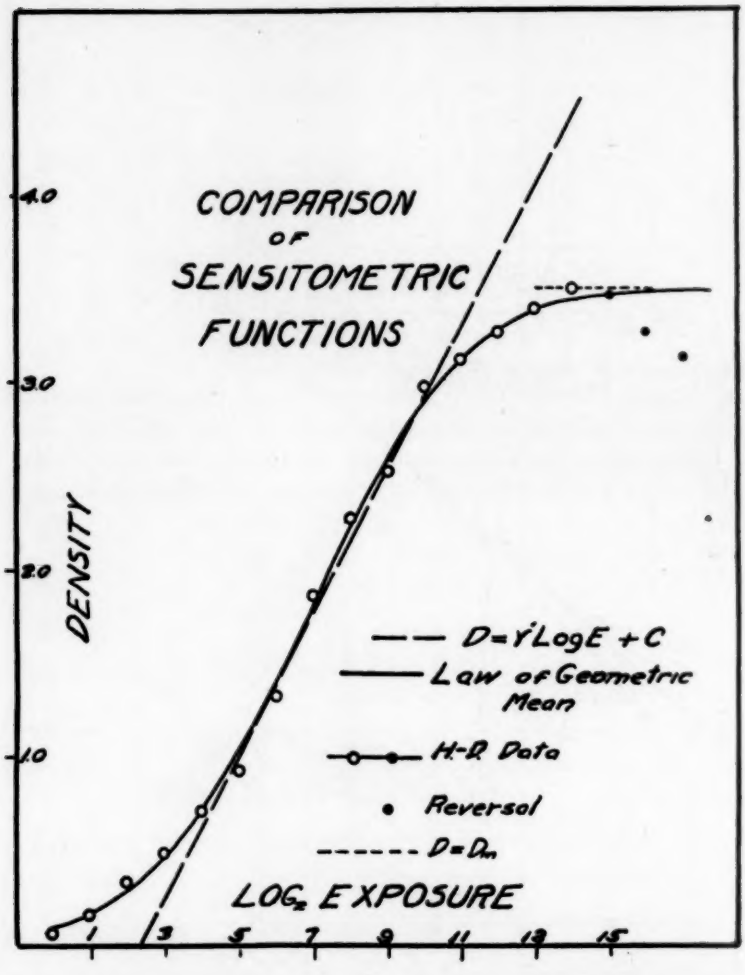

SENSITIVITY
OF A PHOTOGRAPHIC
EMULSION
H-D DATA
LAW OF THE
GEOMETRIC MEAN
E=0
E>0
SCALE OF S
% MAXIMUM DENSITY
LOG₂ EXPOSURE

FIG. 2.


COMPARISON
OF
SENSITOMETRIC
FUNCTIONS
4.0
3.0
2.0
1.0
DENSITY
D=γ'LogE+C
Law of Geometric Mean
H-D Data
Reversal
D=Dm
LOG E EXPOSURE
1
3
5
7
9
11
13
15

acterize the threshold or fog characteristics of the emulsion. If D were strictly proportional to a number of events called out by exposure E, the probability of these events might be viewed as governed by a normal distribution of entropies.

For constant intensity this probability as measured by D tends to a maximum with time, the same being true when for constant duration, intensity is increased. The failure of the reciprocity law could be studied in the light of equation (2), making various hypotheses as to the form of its argument with respect to intensity and time.

Whatever fruit such speculations may bear, the fact will remain that the generalized law of the geometric mean is a useful empirical formula for the behavior of photographic emulsions. The complexity of its form is relieved by the use of probability paper and the fact that in the mid-range of densities it reduces to the conventional $D - \log E$ relation.

REFERENCES.

1. HERTZSPRUNG, A., *Z. f. Wiss. Phot.*, 3, 468, 1905.
2. NEBLETTE, "Photography," p. 235. Ed. 1927.
3. LOVELAND AND TRIVELLI, *J. F. I.*, August 1927.

Dealing Double Death to Flies.—(U. S. D. A. Clip Sheet, No. 820.) The U. S. Department of Agriculture points out that most of the liquid household insecticides now on the market contain insect poisons known as the pyrethrins, which have been obtained from pyrethrum flowers and put into solution in kerosene. Kerosene itself has some action on house flies, but the major effect is produced by the pyrethrins in it. Flies that come in contact with mist from a phrethrum-kerosene spray immediately "nose-dive" to the floor. They appear to be dead, but really are only paralyzed. Some of the paralyzed flies die, but others recover later and become as capable of annoyance as they were before.

The Department is vitally interested in not only how to knock flies down but to keep them down. It was natural that the effect of derris also be determined. Derris, a woody vine related to our locust tree, grows wild and is also cultivated in the East Indies. The immediate effect of derris was not so spectacular as that of pyrethrum. The flies remained in the air longer and offered more resistance to "taking the count," but once they were down they remained down and in the end derris killed a larger percentage of the sprayed flies than did pyrethrum. An insecticidal spray containing both derris and pyrethrum should exert the "knock-down" effect of the latter and the "drag-out" of the former. C.

Lightning Not Dangerous to Aircraft in Flight.—A special subcommittee of the National Advisory Committee for Aeronautics consisting of recognized authorities on meteorological and electrical conditions finds that hazards to aircraft due to electrical phenomena are very slight. There was evidence presented showing that an airship in flight may accumulate an electrostatic charge, but in no reviewed case was this charge shown to be dangerous. It was agreed that the metal framework of either an airplane or an airship presents a high degree of internal protection against the effects of lightning discharges.

A number of reports of damages by lightning to airplanes in flight were reviewed. On the whole, the subcommittee believes it is possible, without serious complication, to render an airplane practically immune to serious danger either to the airplane or to the occupants. There is also small probability that serious damage will result from discharges when trailing radio antenna and cables are reeled in and when the customary protective methods inherent in bonded metallic cage construction are utilized. Of course, all aircraft must make every effort to avoid thunderstorms because of the exceedingly violent and extremely turbulent winds, in themselves most dangerous destructive forces. C.

LATERAL VIBRATION OF RING-SHAPED FRAMES.

BY

FRED H. BROWN,[1] B.M.E., M.S.

One of the major trends of the present age in machine design is in the elimination of noise, particularly in those machines with which the public has daily contact. A large proportion of such machines are driven by electric motors. As much of the noise in alternating current motors comes from vibrating structural members, it is necessary that the designer be able to predict in advance whether or not any part of the machine will be in resonance with any of the alternating forces which are inherently present.

For most of the machine parts the problem of solving for such natural frequencies has been outlined. One of the main elements, however, has been difficult to analyze. This part is the frame.

The frames of many electrical machines have the appear-

FIG. 1.

Frame of electric machine and the shape of its vibration in its own plane.

ance of Fig. 1a and may be regarded as an arc of a circle with built-in ends as in Fig. 1b. The lowest natural frequency of

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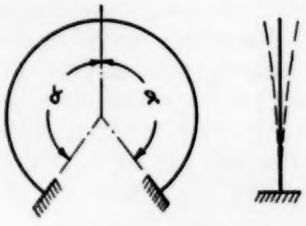
such a frame is for motion in the plane of the ring (Fig. 1c), as discussed by Den Hartog.[2]

It is the purpose of the author to discuss another form of motion which has been found to exist, namely, a combined flexural and torsional vibration perpendicular to the plane of the ring. From the side such a motion has the appearance of a vibrating cantilever beam, as shown in Fig. 2. As the exact solution of this problem is too difficult, an approximate

FIG. 2.



Shape of lateral vibration.

solution based on a modification of the method of Rayleigh [3] is presented.

The calculation of the natural frequency is given in the appendix, the result being

$$f = \frac{1}{2\pi}\psi\left(\alpha, \frac{EI}{C}\right)\sqrt{\frac{EI}{\rho l^4}}, \qquad (1)$$

where

f = natural frequency in cycles per second,
EI = bending rigidity of ring in pounds inches^2,
ρ = mass per inch or arc of the ring,
l = total length of arc in inches,
α = half angle of the arc,
C = torsional rigidity of the ring (a function of the shear modulus, and the shape of the section).[4]

---

[2] J. P. Den Hartog, "Vibration in Frames of Electrical Machines," *Trans. A. S. M. E.*, APM-50-6, 1928.

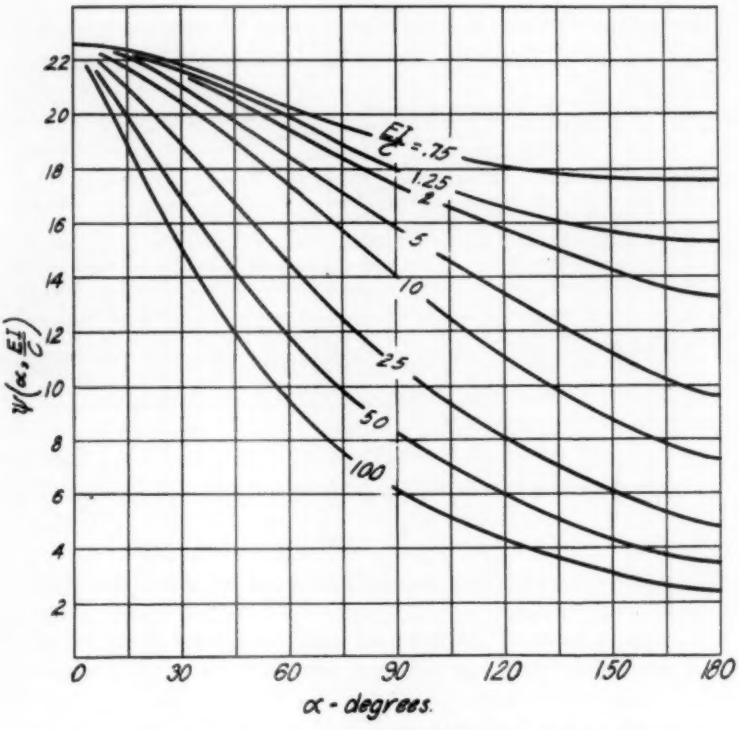
[3] Lord Rayleigh, "Theory of Sound," 2d ed., Vol. 1, pp. 111 and 287.

[4] S. Timoshenko, "Strength of Materials," Part I, p. 76.

The function $\psi(\alpha, EI/C)$ can be taken from either Fig. 3 or Fig. 4, according to which is the more convenient.

Since the method of deriving Eq. 1 is approximate, the frequency was also determined experimentally by means of an accurately calibrated stroboscope. Steel rings of two

FIG. 3.



Frequency coefficient of Eq. (1) vs. semi-central angle for the motion of Fig. 2.

different cross-sections were tested; one round and one rectangular.

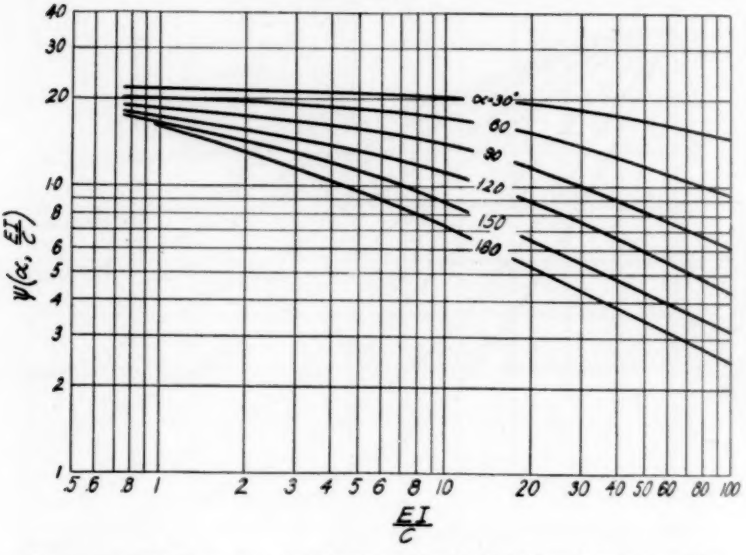
(*a*) The round cross-section ($EI/C = 1.25$, dia. = 1/4 in., length $l = 41.65$ in.) was tested for several values of α from 30° to 150° with results as shown in Table 1.

(*b*) The ring of rectangular cross section ($1/16'' \times 1/2''$, $l = 41.0$ in.) was set up so as to vibrate in the direction of greater stiffness. A simple calculation gives $EI/C = 44$.

TABLE 1.

α, deg.	30.	60.	90.	120.	150.
$f_{calculated}$	18.5	17.1	15.4	14.2	13.4
$f_{observed}$	18.0	17.0	15.8	14.1	13.5
Per cent. error	+2.8	+.6	−2.5	+.7	−.7

FIG. 4.



Frequency coefficient vs. the ratio of bending and torsional stiffness of the cross-section.

A comparison of calculated and observed frequencies is given in Table 2 along with a comparison of the observed and calculated frequency (f') for the flexural vibration in the plane of the ring, as indicated in Fig. 1c.

TABLE 2.

α, deg.	60.	90.	120.	150.	
$f_{calculated}$	34.8	24.6	18.4	13.1	(Fig. 2)
$f_{observed}$	34.0	25.9	18.7	13.0	
Per cent. error	+2.3	−5.0	−1.6	+.7	
$f'_{calculated}$	17.9	14.9	12.0	9.55	(Fig. 1c)
$f'_{observed}$	17.7	14.7	11.9	9.5	
Per cent. error	+1.1	+1.4	+.9	+.5	

The calculated values for the flexural vibration f' are all greater than the observed ones. This is to be expected, due to the fact that the Rayleigh method was used in their determination. Since these calculated values are within the limits of expected error, the values of the constants E, I, l, ρ and α must be sufficiently accurate for the determination of the frequencies in lateral vibration. In the case of the lateral vibration of Fig. 2, it is seen that the calculated values are sometimes higher and sometimes lower than their observed equivalents. On account of the fact that the method of calculation is a *modification* of the Rayleigh method, this was to be expected.

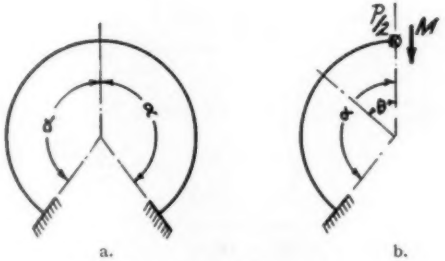
In view of the good agreement of the calculated values with the observed results, Eq. 1 can be considered as sufficiently accurate for most engineering calculations.

APPENDIX.

The method used for obtaining Eq. 1 will now be discussed in detail. As with Rayleigh's method, the frequency was found by equating the maximum potential energy of the vibrating system to its maximum kinetic energy. Rayleigh's method used *one* assumed configuration of the system for the calculations of both energies. In the present case, the execution of the ordinary Rayleigh method, i.e. the use of the *same* assumed curve for the shape calculation of the two energies, led to great complications, either in the algebraic manipulations or in the numerical computations which followed.

Therefore, two different curves were used for the calcula-

FIG. 5.



a. b.

Calculation of potential energy.

tion of these energies. For the potential energy, the static deflection curve of the ring, caused by a concentrated load P at the mid-point of the arc (Fig. 5a) was used. This curve takes account of the potential energies of both twist and flexure. For the purpose of finding the kinetic energy, a simple curve (Eq. 5) was taken, and since the kinetic energy of rotation is very much smaller than that of translation, the former was neglected.

Calculation of Potential Energy.

Isolating one portion of the ring as in Fig. 5b, we find the force $P/2$ and the moment M acting at the cut. At any point (θ) on the ring the bending moment is

$$M_b = \frac{PR}{2}\sin\theta - M\cos\theta,$$

and the twisting moment is

$$M_t = \frac{PR}{2}(1 - \cos\theta) - M\sin\theta.$$

Applying the method of Castigliano,[5] the moment M is found in terms of P as

$$PR\left(\frac{k-1}{4}\sin^2\alpha + \frac{k}{2}\cos\alpha - \frac{k}{2}\right) + M\left(\frac{k+1}{2}\alpha - \frac{k-1}{4}\sin 2\alpha\right) = 0, \quad (2)$$

where $k = EI/C$. Also, the axial deflection δ is found to be

$$\delta = \frac{R^2}{EI}\left[\frac{3k+1}{4}\alpha - k\sin\alpha + \frac{k-1}{8}\sin 2\alpha\right]PR + \left[\frac{k-1}{2}\sin^2\alpha + k\cos\alpha - k\right]M. \quad (3)$$

Substituting the value of M from Eq. 2 in Eq. 3, we obtain:

$$\delta = \frac{PR^3}{EI}\left[F - \frac{AD}{B}\right],$$

[5] S. Timoshenko, loc. cit., p. 318.

where

$$F = \frac{3k+1}{4}\alpha - k\sin\alpha + \frac{k-1}{8}\sin 2\alpha, \quad \text{(From Eq. 3)}$$

$$A = \frac{k-1}{4}\sin^2\alpha + \frac{k}{2}\cos\alpha - \frac{k}{2}, \quad \text{(From Eq. 2)}$$

$$B = \frac{k+1}{2}\alpha - \frac{k-1}{4}\sin 2\alpha, \quad \text{(From Eq. 2)}$$

$$D = 2A = \frac{k-1}{2}\sin^2\alpha + k\cos\alpha - k. \quad \text{(From Eq. 3)}$$

Then

$$P = \frac{EI\delta}{R^3}\frac{B}{BF - AD},$$

whence

$$\text{the potential energy} = \frac{P\delta}{2} = \frac{EI\delta^2}{R^3}\frac{B}{BF - AD}.$$

Calculation of Kinetic Energy.

In order to get the kinetic energy, the following configuration of the system is assumed:

$$y = \frac{\delta}{2}\left(1 - \cos\frac{2\pi\alpha}{\theta}\right). \tag{5}$$

With this expression the kinetic energy of translation calculates to

$$\text{kinetic energy} = \frac{3}{8}\rho\omega^2\delta^2 R\alpha. \tag{6}$$

Equating Eq. 4 and Eq. 6 and solving for ω^2,

$$\omega^2 = \frac{4}{3}\frac{EI}{\rho R^4\alpha^4}\frac{B\alpha^3}{BF - AD}.$$

But $R^4\alpha^4 = (l/2).^4$ Hence

$$\omega^2 = \frac{EI}{pl^4}\frac{64B\alpha^3}{3(BF - AD)}$$

and

$$f = \frac{1}{2\pi}\sqrt{\frac{EI}{\rho l^4}}\,\psi\left(\alpha, \frac{EI}{C}\right), \qquad (1)$$

where

$$\psi\left(\alpha, \frac{EI}{C}\right) = \sqrt{\frac{64B\alpha^3}{3(BF - AD)}}\,.$$

For $\alpha = 0$, i.e. a "clamped-clamped" beam, this expression assumes the value:

$$f = \frac{22.6}{2\pi}\sqrt{\frac{EI}{\rho l^4}}\,,$$

which is but one per cent. higher than the exact solution.

COMPREHENSIVE THEORY OF A POWER FACTOR BRIDGE.

BY

J. C. BALSBAUGH * and ALFRED HERZENBERG.†

The purpose of this paper is to give the complete theory of a bridge that may be used for high-precision power-factor measurements. The subject matter includes a general description of the bridge, general bridge and shield equations, calculation of measured power-factor by the bridge, factors affecting the precision and accuracy of power-factor measurements, sensitivity of the bridge expressed in terms of the minimum detectable changes in power factor and capacitance, detecting circuit for balancing bridge and shield, method of obtaining a bridge balance, calculation of measured capacitance by bridge for obtaining dielectric constants, effect of shield unbalance on the power-factor and capacitance balance of the bridge, method of checking measured bridge power factor, calculation and measurement of the power factor of an oil sample, and the evaluation of the power factor of air condensers.

A companion paper [1] deals specifically with the application of this theory to the measurement of the power factor of small oil samples and includes the details of construction of condensers and oil cells, general characteristics of the bridge, operation of the bridge for power-factor and capacitance measurements, characteristics of oil cells, and the procedure for measuring the power factor of small oil samples.

I. GENERAL DESCRIPTION OF BRIDGE.

A schematic diagram of the bridge with electrostatic shielding at bridge and ground potentials and operation switches is shown in Fig. 1. The component parts of the

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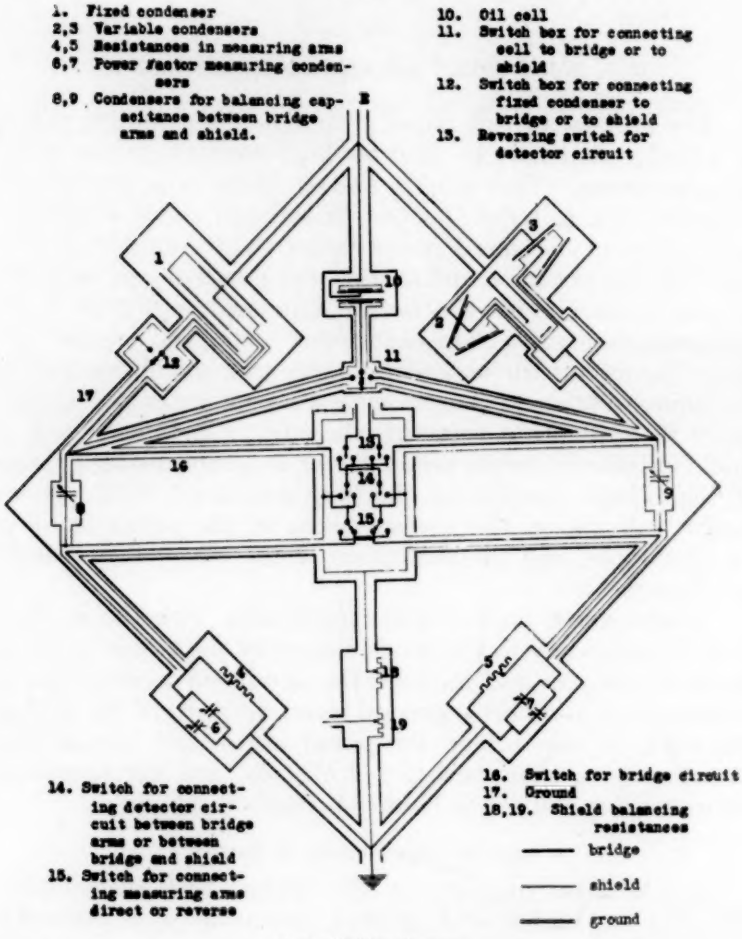
† Research Assistant, Elec. Eng. Dept., Mass. Inst. of Tech.

[1] "The M. I. T. Power-Factor Bridge and Associated Oil Cell, and the Power-Factor Measurement of Small Oil Samples," by J. C. Balsbaugh, N. D. Kenney and A. Herzenberg, A. I. E. E., 1934.

bridge and shield circuits are indicated and explained in Fig. 1. This bridge is of the Schering type; but with symmetrical measuring arms, a variable condenser in one of the high-

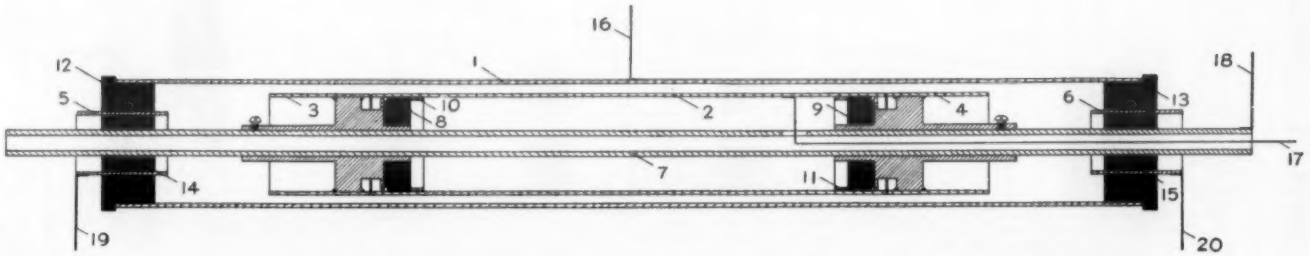
FIG. 1.


1. Fixed condenser
2,3 Variable condensers
4,5 Resistances in measuring arms
6,7 Power factor measuring condensers
8,9 Condensers for balancing capacitance between bridge arms and shield.
10. Oil cell
11. Switch box for connecting cell to bridge or to shield
12. Switch box for connecting fixed condenser to bridge or to shield
13. Reversing switch for detector circuit
14. Switch for connecting detector circuit between bridge arms or between bridge and shield
15. Switch for connecting measuring arms direct or reverse
16. Switch for bridge circuit
17. Ground
18,19. Shield balancing resistances
bridge
shield
ground
E

General bridge diagram.

voltage arms, and variable condensers for balancing the capacitance between the bridge arms and shield.

The fixed and variable condensers (1, 2, and 3 in Fig. 1) are shown in section in Figs. 2 and 3, respectively. These

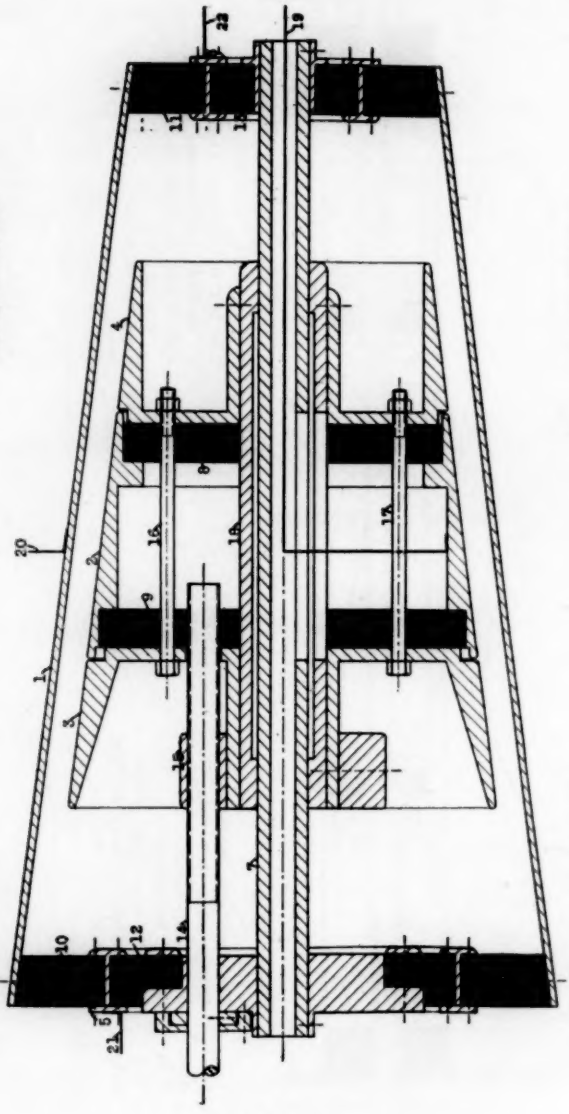


Fixed condenser.

FIG. 2.

1. High voltage section
2. Bridge section
3,4. Shield sections
5,6. Ground sections
7. Shield supporting shaft
8,9. Insulation between bridge and shield section
10,11. Spacers for 3,4 and 8,9.
12,13. Insulation between high voltage section and ground
14,15. Insulation between shield and ground
16. High voltage lead
17. Bridge lead
18. Shield lead
19,20. Ground leads

FIG. 3.

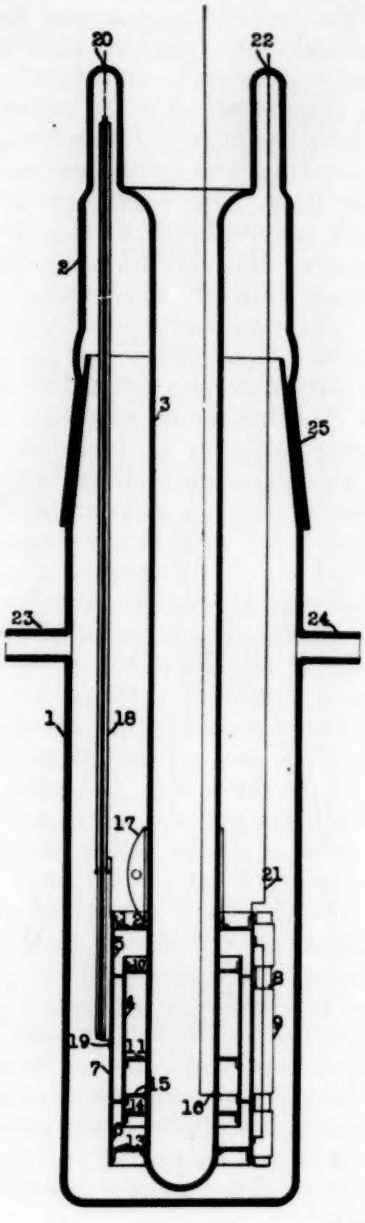

1. High tension section
2. Bridge section
3,4. Shield sections
5,6. Ground sections
7. Shield supporting shaft
8,9. Insulation between bridge and shield
10,11. Insulation between high-voltage and ground
12,13. Insulation between shield and ground
14,15. Shaft and block
16,17. Assembly bolts for 2,3,4.
18. Support for 2,3,4.
19. Bridge lead
20. High voltage lead
21,22. Ground leads

Variable condenser.

condensers are essentially the same in principle but differ in that the fixed condenser is of the cylindrical type and the variable condensers are of the conical type. Also the variable condensers are designed so that through a crank and threaded shaft the capacitance of these condensers may be varied. These condensers are designed with insulation between bridge and shield sections, between high-voltage and ground sections, and between shield and ground sections. The use of a ground between high-voltage and shield prevents variable surface leakage and conduction currents of the insulation for the high-voltage section from giving an unsteady shield balance. It is also important to have the insulation between bridge and shield effectively shielded from the high-voltage section to prevent any loss entering the bridge through this insulation. It is shown theoretically in Part VIII that the bridge measurements of power factor and capacitance may be made independent of the power factor of the insulation between bridge and shield. This fact may also be checked experimentally by taking bridge measurements with and without a resistance connected, between bridge and shield.

Two types of oil cells which may be used for precision power-factor and capacitance measurements on small oil samples are shown in Figs. 4 and 5. The cell shown in Fig. 4 is constructed with a ground joint and with all of the electrical measuring surfaces mounted on a supporting tube so that the measuring surfaces may be removed from the assembled cell and effectively cleaned. For the cell in Fig. 4 the shield for the measuring section is placed outside the cell (may be made a part of the heating oven in which the cell is placed) so as to reduce the oil volume required for the cell and to reduce the number of surfaces to be cleaned. The cell shown in Fig. 5 is in principle the same as the cell in Fig. 4 except that the shield for the measuring section is placed within the cell and also that a copper tube is sealed into the glass tube through which the measuring section lead is brought out of the cell. The use of this copper tube connected to shield prevents surface leakage and conduction currents through the glass from the high-voltage lead and cylinders from entering the bridge section. The electrical measuring surfaces, supports, and mounting are made of nickel and the assembly

FIG. 4.



1. Pyrex glass container
2. Pyrex glass cap
3. Pyrex glass supporting tube for electrical measuring surfaces
4. High voltage section
5,6. Shield sections
7. Measuring section
8. Pyrex glass rod for supporting 5,6,7
9. Nickle band for assembling 5,6,7,8.
10,11,12,13,14. Nickle disks for supporting 4,5,6,7 on supporting tube 3
15. Support for 4
16. Lead to 4
17. Support for assembly of 5,6,7
18. Shield for 19
19. Lead to 7
20 Seal for 19
21 Lead to shield
22 Seal for 21
23,24 Tubulations
25 Ground joint

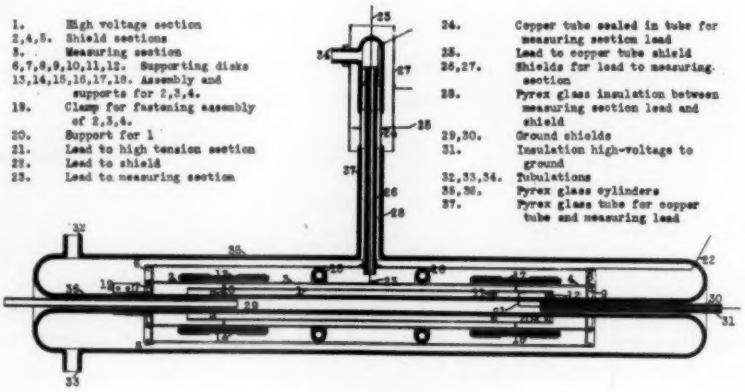
Oil cell.

of these surfaces, supports, and leads is accomplished through spot-welding.

II. GENERAL BRIDGE EQUATIONS.

The general circuit diagram of the bridge is shown in Fig. 6 in which b_1 and b_2 are the points of the bridge arms, s represents the shield, Z_1, Z_2, Z_7, and Z_8 the bridge arm impedances, Z_3 the impedance between the high-voltage and shield, Z_6 the

FIG. 5.


1. High voltage section
2,4,5. Shield sections
3. Measuring section
6,7,8,9,10,11,12. Supporting disks
13,14,15,16,17,18. Assembly and supports for 2,3,4.
19. Clamp for fastening assembly of 2,3,4.
20. Support for 1
21. Lead to high tension section
22. Lead to shield
23. Lead to measuring section
24. Copper tube sealed in tube for measuring section lead
25. Lead to copper tube shield
26,27. Shields for lead to measuring section
28. Pyrex glass insulation between measuring section lead and shield
29,30. Ground shields
31. Insulation high-voltage to ground
32,33,34. Tubulations
35,36. Pyrex glass cylinders
37. Pyrex glass tube for copper tube and measuring lead

Oil cell.

shield balancing impedance, Z_9 the galvanometer impedance or input impedance of an amplifier, Z_4 and Z_5 the impedances between the bridge arms and shield, and E the applied bridge voltage.

Solving this general circuit diagram for I_9 gives

$$I_9 = \frac{EN}{D}, \tag{1}$$

where

$$N = Z_2Z_7Z_3Z_4Z_5Z_6\left(\frac{Z_1}{Z_3Z_4} + \frac{Z_8}{Z_5Z_6} + \frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5} + \frac{1}{Z_6}\right)$$

$$- Z_1Z_8Z_3Z_4Z_5Z_6\left(\frac{Z_2}{Z_3Z_5} + \frac{Z_7}{Z_4Z_6} + \frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5} + \frac{1}{Z_6}\right) \tag{2}$$

and

$$D = Z_3Z_4Z_5Z_6\left(\frac{1}{Z_3}+\frac{1}{Z_4}+\frac{1}{Z_5}+\frac{1}{Z_6}\right)[Z_1Z_2(Z_7+Z_8+Z_9)$$
$$+ Z_7Z_8(Z_1+Z_2+Z_9)+Z_9(Z_1Z_8+Z_2Z_7)]$$
$$+ Z_2Z_3Z_4Z_6Z_8Z_9(Z_1+Z_7)\left(\frac{1}{Z_3}+\frac{1}{Z_4}+\frac{1}{Z_6}\right)$$
$$+ Z_1Z_3Z_5Z_6Z_7Z_9(Z_2+Z_8)\left(\frac{1}{Z_3}+\frac{1}{Z_5}+\frac{1}{Z_6}\right)$$
$$+ Z_1Z_2Z_7Z_8(Z_3+Z_6)(Z_4+Z_5+Z_9). \qquad (3)$$

An inspection of (1), (2), and (3) will show that I_9 will be equal to zero for any two of the following conditions:

$$Z_1Z_8 = Z_2Z_7, \qquad (4)$$

$$Z_1Z_6 = Z_3Z_7, \qquad (5)$$

$$Z_2Z_6 = Z_3Z_8. \qquad (6)$$

If any two of (4), (5), and (6) hold, then the remaining one will also hold. Thus in Fig. 6 the current I_9 will be equal to zero for the condition of b_1, b_2, and s being at the same potential. It is furthermore interesting and important to note from (1) that even if (5) and (6) do not hold, that is, the shield is unbalanced, but if (4) and

$$\frac{Z_4}{Z_5}=\frac{Z_1}{Z_2} \qquad (7)$$

hold, the current I_9 will still be zero.

If the bridge circuit is considered by itself, that is, for the condition of

$$Z_4 = Z_5 = \infty, \qquad (8)$$

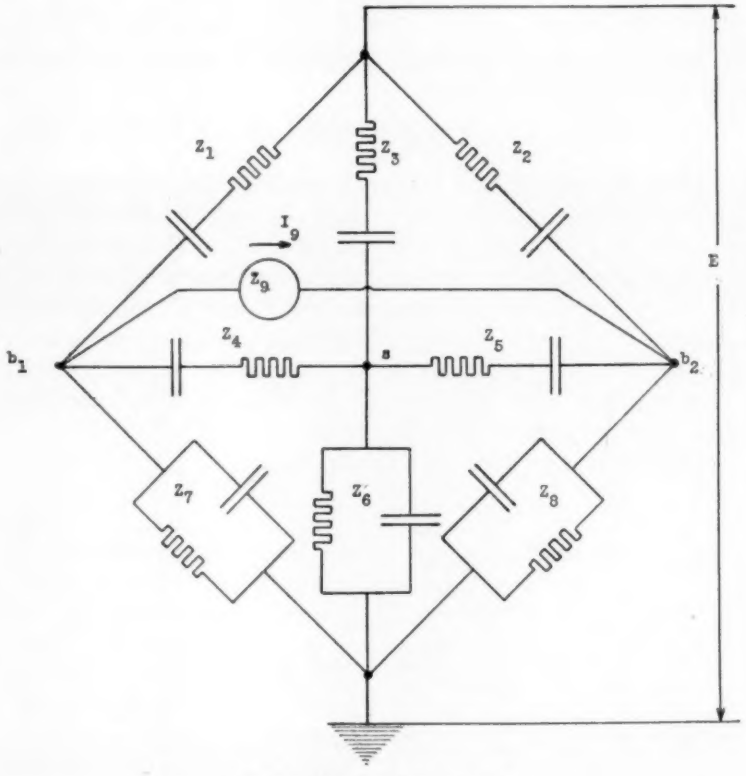
then the current through Z_9 becomes

$$I_9' = \frac{E(Z_2Z_7 - Z_1Z_8)}{Z_1Z_2(Z_7+Z_8+Z_9)+Z_7Z_8(Z_1+Z_2+Z_9)+Z_9(Z_2Z_7+Z_1Z_8)}. \qquad (9)$$

While (1) and (9) give the exact value of I_9 for the conditions assumed, it is sometimes desirable to use approximate equations. Thus I_9' in (9) may be approximated for $Z_7 = Z_8$ by

$$I_9' = E\frac{(Z_2 - Z_1)}{Z_1 Z_2}\frac{Z_7}{2Z_7 + Z_9} \qquad (10)$$

FIG. 6.



Equivalent bridge and shield circuits.

and I_9 from (1) may be approximated for $Z_7 \cong Z_8$ by

$$I_9 = E\left(\frac{Z_7}{2Z_7 + Z_9}\right)\left[\frac{Z_2 - Z_1}{Z_1 Z_2} + \frac{(Z_6 Z_1 - Z_7 Z_3)(Z_5 - Z_4)}{Z_1 Z_3 Z_4 Z_5}\right]. \qquad (11)$$

III. CALCULATION OF MEASURED POWER FACTOR BY BRIDGE.

A solution of the network in Fig. 6 will show that the galvanometer current will in general be equal to zero for

$$\frac{Z_1}{Z_2} = \frac{Z_7}{Z_8}\frac{\left[\begin{array}{l} Z_1Z_5Z_6 + Z_3Z_4Z_8 \\ \qquad + Z_3Z_4Z_5Z_6\left(\frac{1}{Z_3}+\frac{1}{Z_4}+\frac{1}{Z_5}+\frac{1}{Z_6}\right)\end{array}\right]}{\left[\begin{array}{l} Z_2Z_4Z_6 + Z_3Z_5Z_7 \\ \qquad + Z_3Z_4Z_5Z_6\left(\frac{1}{Z_3}+\frac{1}{Z_4}+\frac{1}{Z_5}+\frac{1}{Z_6}\right)\end{array}\right]}. \tag{12}$$

Assuming that the shield is balanced, that is, (5) and (6) hold, then (12) reduces to

$$Z_1Z_8 = Z_2Z_7. \tag{13}$$

Thus for shield and bridge balanced, the resistance and capacitance of impedances Z_1 and Z_2 may be evaluated in terms of Z_2 or Z_1, respectively, Z_7 and Z_8. This shows that it is not necessary to evaluate the impedances Z_3 and Z_6 of the shield circuit and the impedances Z_4 and Z_5 for the determination of either Z_1 or Z_2. Therefore one of the reasons for the use of a shield circuit is to permit the measurement of an impedance such as Z_1 or Z_2 in terms of other constants which can be accurately determined.

The impedances Z_1, Z_2, Z_7, and Z_8 are

$$Z_1 = R_1 - j\frac{1}{\omega C_1}, \tag{14}$$

$$Z_2 = R_2 - j\frac{1}{\omega C_2}, \tag{15}$$

$$Z_7 = \frac{R_7k_7 - j\omega R_7^2k_7^2C_7}{1 + \omega^2R_7^2k_7^2C_7^2}, \tag{16}$$

$$Z_8 = \frac{R_8k_8 - j\omega R_8^2k_8^2C_8}{1 + \omega^2R_8^2k_8^2C_8^2}, \tag{17}$$

where

$$k_7 = \frac{R_{c7}}{R_7 + R_{c7}}, \tag{18}$$

$$k_8 = \frac{R_{c8}}{R_8 + R_{c8}}. \tag{19}$$

R_{c7} and R_{c8} represent resistances in shunt with C_7 and C_8, respectively, so that the effect of losses in the power-factor-measuring capacitances may be taken into account. Substituting Z_1, Z_2, Z_7, and Z_8 from (14), (15), (16), and (17), respectively, in (13) and solving for R_2 and C_2 gives

$$R_2 = \frac{R_8 k_8}{C_1 R_7 k_7}\left[\frac{R_7 k_7 C_7 - R_8 k_8 C_8 + R_1 C_1 + \omega^2 R_1 R_7 R_8 k_7 k_8 C_1 C_7 C_8}{1 + \omega^2 R_8^2 k_8^2 C_8^2}\right], \quad (20)$$

$$C_2 = \frac{R_7 k_7 C_1}{R_8 k_8}\left[\frac{1 + \omega^2 R_8^2 k_8^2 C_8^2}{1 + \omega^2 R_1 C_1 (R_8 k_8 C_8 - R_7 k_7 C_7) + \omega^2 R_7 R_8 k_7 k_8 C_7 C_8}\right]. \quad (21)$$

The power factor p_{f2} of impedance Z_2 is given by

$$p_{f2} = \frac{R_2}{Z_2} = \frac{R_2}{\left(R_2^2 + \frac{1}{\omega^2 C_2^2}\right)^{1/2}} = \frac{\omega R_2 C_2}{(1 + \omega^2 R_2^2 C_2^2)^{1/2}}. \quad (22)$$

Let

$$\omega R_2 C_2 = p_2 = \frac{R_2}{X_2}, \quad (23)$$

where p_2 is the tangent of the imperfection angle of Z_2. Then the power factor p_{f2} of Z_2 from (22) becomes

$$p_{f2} = \frac{p_2}{\sqrt{1 + p_2^2}}. \quad (24)$$

In general in this work, power factors will be expressed in terms of p as in (23), that is, the tangent of the angle equal to 90° minus the power-factor angle. Where the power factor is desired to a greater accuracy than as given by p, it may be obtained from (24) or from the sine of the angle whose tangent is given by p.

For low values of power factor, $\omega R_2 C_2$ or p_2, is small in comparison with unity so that

$$p_{f2} = \cos \theta_2 \cong p_2. \quad (25)$$

The difference between p_{f2} and p_2 for relatively low power factors may be closely approximated by

$$p_{f2} \cong p_2 - \tfrac{1}{2} p_2^3 \quad (26)$$

and the difference between p_{f2} and p_2 will be less than a given power factor p_p for the condition of p_2 equal to approximately $\sqrt[4]{2p_p}$. Thus the difference between p_2 and p_{f2} as given by (23) and (22), respectively, will be less than 10^{-6} for p_2 of the order of one per cent.

Substituting R_2 from (20) and C_2 from (21) in (23) gives

$$p_2 = \frac{\omega R_7 k_7 C_7 - \omega R_8 k_8 C_8 + \omega R_1 C_1 + \omega^3 R_1 R_7 R_8 k_7 k_8 C_1 C_7 C_8}{1 + \omega^2 R_1 C_1 (R_8 k_8 C_8 - R_7 k_7 C_7) + \omega^2 R_7 R_8 k_7 k_8 C_7 C_8}. \quad (27)$$

Now if we assume that the power factors of the measuring capacitances C_7 and C_8 are equal to zero, then k_7 and k_8 are equal to unity. Also with C_7 and C_8 relatively low so that $\omega R_7 C_7$ and $\omega R_8 C_8$ are very small in comparison to unity, then (27) becomes

$$p_2 = \omega R_7 C_7 - \omega R_8 C_8 + \omega R_1 C_1. \quad (28)$$

However, $\omega R_1 C_1$ from (23) is the tangent of the imperfection angle of Z_1 and

$$p_1 = \omega R_1 C_1. \quad (29)$$

Then from (28)

$$p_2 = \omega R_7 C_7 - \omega R_8 C_8 + p_1. \quad (30)$$

Expressing R_7, C_7, and R_8, C_8 similar to p_2 and p_1, from (23) gives

$$p_2 = \frac{1}{p_7} - \frac{1}{p_8} + p_1. \quad (31)$$

Equation (30) shows that the power factor of impedance Z_2 is given in terms of resistances and capacitances of the low-tension bridge arms and also the power factor of the impedance Z_1. This means that it is necessary to investigate the power factor of the condenser forming Z_1 in the bridge and determine if this power factor is significant in the range to which it is desired to express p_2.

It is obviously pertinent to investigate the difference between p_2 as given by (27) and (30). Any difference between these two equations will be determined principally from the magnitudes of C_7 and C_8 and the power factors of the

measuring capacitances. Equation (27) may be written as

$$p_2 = \frac{\begin{array}{r}(m_7 - m_8) + m_7 m_8(p_{c8} - p_{c7}) \\ + p_1[(m_7 p_{c7} + 1)(m_8 p_{c8} + 1) + m_7 m_8]\end{array}}{\begin{array}{r}(m_7 p_{c7} + 1)(m_8 p_{c8} + 1) + m_7 m_8 \\ + p_1[(m_8 - m_7) + m_7 m_8(p_{c7} - p_{c8})]\end{array}}, \quad (32)$$

where p_{c7} and p_{c8} are the tangents of the imperfection angles of C_7 and C_8 and are

$$p_{c7} = \frac{1}{\omega R_{c7} C_7}, \quad (33)$$

$$p_{c8} = \frac{1}{\omega R_{c8} C_8}, \quad (34)$$

and

$$m_7 = \omega R_7 C_7, \qquad m_8 = \omega R_8 C_8. \quad (35)$$

In Table I there is given the difference between p_2 as calculated from (32) and p_2 as calculated from (30) for different values of p_1, p_{c7}, p_{c8}, and $(m_7 - m_8)$. Thus with $(m_7 - m_8)$

TABLE I.

p_2 from (32) − p_2 from (30) (values times 10^{-6}).

			m_8					
			0	10^{-4}	10^{-3}	10^{-2}	10^{-1}	0.5
p_1	$p_{c7} = p_{c8}$	$m_7 - m_8$						
0 or 10^{-4}	0	10^{-5}						−2
"	0	10^{-4}					−1	−20
"	0	10^{-2}				−2	−109	−2032
"	10^{-2}	10^{-5}						−2
"	10^{-2}	10^{-4}					−1	−21
"	10^{-2}	10^{-2}	−1	−1	−1	−5	−129	−2096

$= 10^{-2}$ or one per cent., $m_8 = 0.1$ (that is with a frequency of 60 cycles per second and $R_8 = 10^4$ ohms, C_8 from (35) equals 26,525.8(10^{-12}) farads), $m_7 = 0.11$ (that is with $R_7 = 10^4$ ohms, C_7 from (35) equals 29,178.4(10^{-12}) farads); and $p_1 = p_{c7} = p_{c8} = 0$, the difference between p_2 from (32) and p_2 from (30) is 109(10^{-6}), with p_2 from (30) being the larger value. The value of p_2 from (32) is 9890(10^{-6}) and that from (30) is 10,000(10^{-6}). The difference between p_2 as calculated from (32) and (30) will depend principally upon the magnitudes of

m_7 and m_8, that is C_7 and C_8, the magnitude of p_1, and the magnitudes of and the differences between p_{c7} and p_{c8}. For large values of m_7 and m_8 the difference between p_2 from (32) and (30) may become quite substantial even though p_1, p_{c7}, and p_{c8} are equal to zero. It is therefore advisable in measuring either relatively large or small power factors that the capacitance of either C_7 or C_8 be maintained relatively small so that the values of p may be obtained from (30).

In determining the measured power factor of the bridge $(p_2 - p_1)$ in (30) it is necessary to determine to the required accuracy the values of frequency, R_7, R_8, C_7, and C_8. It is obvious that any of the usual methods may be used in determining the frequency, R_7 and R_8, since a given percentage error in determining these values will give the same percentage error in $(p_2 - p_1)$, for the condition of equal ratio arms, that is, $R_7 = R_8$. When $R_7 \neq R_8$, the error in the calculated power factor will depend upon the errors in the determination of both R_7 and R_8 and also upon the magnitudes of C_7 and C_8. Therefore for $R_7 = R_8$, it is only in determining relatively large values of $(p_2 - p_1)$ to a relatively high power-factor precision that it is necessary to determine quite accurately the values of frequency and R_7 or R_8. R_7 may be adjusted to be very closely equal to R_8 through reversal of the bridge arms as explained in the following.

The determination of the values of C_7 and C_8 in (28) is, however, more difficult. In general C_7 and C_8 will include capacitances connected in arms 7 and 8 of the bridge, respectively, and also shunt capacitances to ground of the resistances R_7 and R_8 and other shunt capacitances. As explained previously, one of the uses of a shield circuit is to permit a relatively simple evaluation of the constants of the impedances Z_1 and Z_2 in the bridge arms. However, since it is impracticable to shield resistances, some other method must be utilized for an accurate determination of the difference between C_7 and C_8. It should be noted that arms 7 and 8 of the bridge circuit should be shielded at ground potential since this will tend to make the bridge balance independent of the shield balance as far as these shunt capacitances are concerned.

Assume that the bridge and shield circuits are first balanced with the bridge arms connected direct in Fig. 6, that

is, arms 7 and 8 are connected to arms 1 and 2, respectively. Then the power factor measured by the bridge from (30) is

$$p_2 - p_1 = \omega R_7 C_7 - \omega R_8 C_{8a}, \tag{36}$$

where C_{8a} is the capacitance to ground in arm 8. Now assume that the bridge arms are reversed, that is, arms 7 and 8 are connected to arms 2 and 1, respectively. Then, assuming a difference between p_2 and p_1 and the capacitance in arm 8 adjusted for balance, gives

$$p_2 - p_1 = \omega R_8 C_{8b} - \omega R_7 C_7. \tag{37}$$

Since in general R_7 and R_8 are not exactly equal, rebalance of the bridge for (32) will require either an adjustment of the relative values of R_7, R_8, or C_1 and C_2. Capacitance rebalance of the bridge may be obtained through a vernier variable condenser in one of the arms 1 or 2. The use of a vernier variable condenser in arms 1 or 2 will permit a closer capacitance balance to be obtained than is practicable through changes in R_7 and R_8. Since the required change in C_1 or C_2 will be of the order of 10^{-5} to 10^{-6} of C_1 or C_2, obviously the change in the absolute power factor of Z_1 or Z_2 (p_1 and p_2, respectively) will be entirely negligible.

Adding (36) and (37) gives for $(p_2 - p_1)$

$$p_2 - p_1 = \omega R \frac{(C_{8b} - C_{8a})}{2}, \tag{38}$$

in which

$$R = R_7 = R_8.$$

Equation (38) shows that the difference in the absolute power factors $(p_2 - p_1)$ is now determined by the difference in the capacitances to ground of arm 8 in the two measurements. Since the shunt capacitance to ground of arm 8 is not changed in the two measurements, this method eliminates the necessity of determining this capacitance. The capacitance $(C_{8b} - C_{8a})$ is then equal to the difference in capacitance of the condenser in arm 8 in the two balances. This difference may be obtained to a greater accuracy than the absolute value for any particular setting. This difference $(C_{8b} - C_{8a})$ may be determined directly by calibration with a standard

condenser or from the dimensions directly of a properly designed capacitance. For measuring power factors in the range of 10^{-6} condensers C_7 and C_8 should be air condensers of a design such that humidity and temperature variations will not produce a measurable effect.

With a condenser similar to the variable condenser in Fig. 3 used as a part of C_7 or C_8, obviously relatively small changes in capacitance may be obtained very easily from a properly designed condenser. Such a variable condenser as a part of C_7 or C_8 should be connected similarly as Item 2 in Fig. 1 except that the high-voltage section would now be connected to ground. This method requires only the evaluation of changes in capacitance to bridge.

The power factor as measured from (38) may also be expressed in terms of larger differences in capacitance $(C_{8b} - C_{8a})$ in (38) by decreasing R, provided sufficient bridge sensitivity is available for these lower values of R.

IV. SENSITIVITY OF BRIDGE.

Assuming first the bridge circuit by itself, that is, neglecting the effect of the shield circuit, or that Z_4 and Z_5 are infinite, then the voltage E_b across the bridge arms from (9) is

$$E_b = I_9' Z_9 = \frac{(Z_2Z_7 - Z_1Z_8)Z_9E}{Z_1Z_2(Z_7 + Z_8 + Z_9) + Z_7Z_8(Z_1 + Z_2Z_9) + Z_9(Z_2Z_7 + Z_1Z_8)} \tag{39}$$

and the voltage E_b will be equal to zero for the condition of $Z_2Z_7 = Z_1Z_8$. Now if Z_1 is changed from balance by ΔZ_1 then

$$\Delta E_b = \frac{dE_b}{dZ_1}\Delta Z_1 \tag{40}$$

and

$$\Delta Z_1 = \Delta R_1 - j\Delta X_1. \tag{41}$$

Expressing ΔR_1 in terms of a change (Δp_1) in the tangent of the imperfection angle or power factor of Z_1 from (23), with a positive Δp_1 being an increase in the power factor of Z_1, and ΔX_1 in terms of a change (Δq_1) equal to the ratio of the change ΔC_1 in capacitance of Z_1 to C_1, with a positive Δq_1 being an

increase in the capacitance of Z_1, then

$$X_1 = \frac{1}{\omega C_1}, \tag{42}$$

$$\Delta X_1 = -\frac{\Delta C_1}{C_1}\frac{1}{\omega C_1} = -\Delta q_1 X_1, \tag{43}$$

and ΔZ_1 in (41) becomes

$$\Delta Z_1 = (\Delta p_1 + j\Delta q_1)X_1. \tag{44}$$

The impedance Z_9 represents the impedance of the galvanometer when a galvanometer is used directly across the bridge arms or the input impedance of the amplifier when an amplifier is used.

Then ΔE_b from (40) becomes

$$\Delta E_b = \frac{dE_b}{dZ_1}(\Delta p_1 + j\Delta q_1)X_1. \tag{45}$$

Equation (45) shows that a given change in the power factor of Z_1, say $\Delta p_1 = 10^{-6}$, gives the same magnitude of ΔE_b or bridge unbalance as a corresponding capacitance change in C_1, that is, $\Delta C_1 = 10^{-6}C_1$. From (39)

$$\frac{dE_b}{dZ_1} = \frac{Z_a(-Z_8Z_9)E}{Z_a^2} - \frac{(Z_2Z_7 - Z_1Z_8)Z_9E\dfrac{dZ_a}{dZ_1}}{Z_a^2}, \tag{46}$$

where Z_a is the denominator of (39).

Since the change is from bridge balance, that is, $Z_2Z_7 = Z_1Z_8$, then

$$\frac{dE_b}{dZ_1} = \frac{-Z_8Z_9E}{Z_1Z_2(Z_7 + Z_8 + Z_9) + Z_7Z_8(Z_1 + Z_2 + Z_9) + Z_9(Z_2Z_7 + Z_1Z_8)}. \tag{47}$$

This gives for ΔE_b in (45)

$$\Delta E_b = \frac{-(\Delta p_1 + j\Delta q_1)X_1Z_8Z_9E}{Z_1Z_2(Z_7 + Z_8 + Z_9) + Z_7Z_8(Z_1 + Z_2 + Z_9) + Z_9(Z_2Z_7 + Z_1Z_8)}. \tag{48}$$

With Z_7, Z_8, and Z_9 of the order of 10^4 ohms and Z_1 and Z_2 equal to approximately $2.65(10^4)$ ohms (corresponding to

C_1 and C_2 equal to $100(10^{-12})$ farads) then obviously Z_7, Z_8, or Z_9 may be neglected in comparison with Z_1 or Z_2. Also assuming approximately equal ratio arms and the following:

$$Z_1 \cong Z_2' \cong X_1 \cong \frac{1}{\omega C_1}, \tag{49}$$

$$Z_7 \cong Z_8 \cong R_7 \cong R_8 = R. \tag{50}$$

ΔE_b from (48) may be written

$$\Delta E_b \cong \omega C_1(\Delta p_1 + j\Delta q_1)EK_b, \tag{51}$$

where

$$K_b = \frac{RZ_9}{2R + Z_9}. \tag{52}$$

Now if we assume that a galvanometer or galvanometer with amplifier is used, which gives one mm. deflection for an input voltage or voltage across the bridge arms equal to E_g, then from (51) the value of Δp_m and Δq_m for one mm. deflection of the galvanometer is equal to

$$\Delta p_m = \Delta q_m \cong \frac{E_g}{\omega C_1 E K_b}. \tag{53}$$

Thus the change of Δp or Δq for a deflection of one mm. of the galvanometer decreases directly as E_g decreases and decreases directly with an increase in frequency, capacitance C_1 or C_2 in high-tension arms of the bridge, the applied bridge voltage E and the value of K_b.

The per-unit magnitude change in K_b for a per-unit change in R for Z_9 being a resistance equal to R_9 is from (52)

$$\frac{\Delta K_b}{K_b} = \frac{\Delta R}{R}\left(\frac{R_9}{2R + R_9}\right) \quad (Z_9 = R_9) \tag{54}$$

and the per-unit magnitude change in K_b for a per-unit change in R_9 is from (52)

$$\frac{\Delta K_b}{K_b} = \frac{\Delta R_9}{R_9}\left(\frac{2R}{2R + R_9}\right) \quad (Z_9 = R_9). \tag{55}$$

Thus the per-unit magnitude change in K_b for a given per-unit change in either R or R_9 will be equal for $R_9 = 2R$.

Similarly for $Z_9 = \pm jX_9$ we get

$$\frac{\Delta K_b}{K_b} = \frac{\Delta R}{R}\left(\frac{X_9^2}{4R^2 + X_9^2}\right), \tag{56}$$

$$\frac{\Delta K_b}{K_b} = \frac{\Delta X_9}{X_9}\left(\frac{4R^2}{4R^2 + X_9^2}\right), \tag{57}$$

and the per-unit magnitude change in K_b for a given per-unit change in either R or $\pm jX_9$ will be equal for $X_9 = 2R$.

This shows that it is desirable to have Z_9 or R_9 larger than R. A study of (52) also shows that K_b will be equal to R for $Z_9 = \infty$ and that for Z_9 in range of practical values of R, K_b will be equal to some fraction of R. Obviously it is desirable to have large values of R (assuming that Z_9 may be varied with R) so that ΔE_b for a given Δp or Δq will be increased or so that Δp_m or Δq_m will be decreased. However, as R is increased, the change in capacitance (C_7 or C_8) for a given value of Δp decreases so that increased difficulty would be encountered in the power-factor measurements. Where the bridge is desired to measure the power factor of small oil samples the value of C_1 is also limited by a convenient size of cell and the volume of oil required for testing.

A study of ΔE_b as in (48) obtained from (1) instead of (9) will give the effect of the shield circuit on the value of Δp_m and Δq_m for given bridge and detecting circuits. This will show that in general, for practical values of Z_3, Z_4, and Z_5 in Fig. 6, the effect of the shield will be to increase the value of ΔE_b for a given Δp or Δq and for given bridge and detecting circuits. For very large values of the capacitances of Z_3, Z_4, and Z_5 in comparison with the capacitance of Z_1 the effect of the shield may decrease the value of ΔE_b for a given Δp or Δq and for given bridge and detecting circuits. In general the effect of the shield on ΔE_b will only be several per cent. (or smaller) of that as given by (51).

V. DETECTING CIRCUIT FOR BALANCING BRIDGE AND SHIELD.

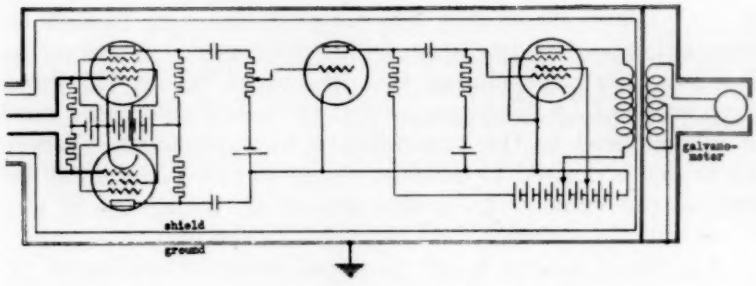
The detector circuit for balancing both the bridge and shield is shown in Fig. 7. The detector circuit consists of a resistance-capacitance coupled amplifier feeding into either a

vibration-type galvanometer or an alternating-current galvanometer with the field energized from a phase shifter.

The complete amplifier is shown shielded at shield and ground potentials. The first stage of the amplifier is connected push-pull, so that the capacitances between shield and each arm of the bridge will be approximately equal. It is also possible to use a single tube for the first stage and then balance the capacitance between shield and either bridge arm by a condenser connected between one of the bridge arms and shield.

The amount of voltage amplification required will depend upon the accuracy to which it is desired to balance, that is,

FIG. 7.



Amplifier and galvanometer.

the value of Δp or Δq which it is desired to be able to detect, upon the voltage ΔE_b appearing across the bridge arms or input voltage to the amplifier for this particular value of Δp or Δq and the voltage sensitivity of the galvanometer. The magnitude of the voltage, ΔE_b, appearing across the bridge arms for a given power factor or capacitance unbalance (Δp or Δq, respectively) of the bridge is given by (51). Then the required voltage amplification will be given by the ratio of ΔE_b from (51), for the desired value of Δp or Δq, to E_s the voltage sensitivity of the galvanometer.

Thus in (51) if we assume that E_1 the bridge voltage is 1000 volts; $K_b = R = 10^4$ ohms, corresponding to the condition of the input impedance of the amplifier, very large in comparison with R; $\omega = 377$, corresponding to a frequency

of 60 cycles per second of the applied voltage; and a value of Δp or Δq equal to 10^{-6}, the value of ΔE_b is equal to $0.94(10^{-7})$ volts. With a galvanometer having a voltage sensitivity of, say, $8(10^{-4})$ volts per mm. deflection, the required amplification is approximately 8500.

VI. BRIDGE BALANCE.

The bridge circuit may be balanced by connecting the detecting circuit across the bridge arms and adjusting C_7 and C_8 and through adjusting either R_7 and R_8 or the variable condensers Z_{21} and Z_{22} (Fig. 12). Of course the bridge and shield must be balanced successively, that is, first connecting the detecting circuit between the bridge arms and then adjusting for approximate bridge balance; then connecting the detecting circuit between either bridge arm and shield and adjusting for approximate shield balance and continuing this procedure until with the detecting circuit successively between the bridge arms and between bridge and shield there is no measurable deflection in the desired range of balance. The number of times it is necessary to continue this procedure of bridge and shield balancing will depend upon the effect of a shield unbalance on the bridge and the effect of a bridge unbalance on the shield. These effects will in general depend upon the coupling between bridge and shield, that is, the magnitude of Z_4 and Z_5 in comparison with Z_1 and Z_2, the degree of unbalance and the difference between Z_4 and Z_5. By adjusting either Z_4 or Z_5 until Z_4 is approximately equal to Z_5 the effect of a bridge unbalance on the shield or the effect of a shield unbalance on the bridge may be made quite small. A complete study of the effect of a shield unbalance on the bridge is given under Part VIII.

If we assume that both bridge and shield circuits are balanced we have by (13)

$$Z_1 Z_8 = Z_2 Z_7.$$

Now if Z_1 is changed by ΔZ_1, rebalance of the bridge may be obtained through a change in Z_8 equal to ΔZ_8 and rebalancing of the shield. For small values of ΔZ_1 and neglecting second-

order effects, from (13)

$$Z_1\Delta Z_8 = -Z_8\Delta Z_1. \tag{58}$$

ΔZ_1 may be expressed from (44) as

$$\Delta Z_1 = (\Delta p_1 + j\Delta q_1)X_1 \tag{59}$$

and

$$\Delta Z_8 = \frac{\partial Z_8}{\partial R_8}\Delta R_8 + \frac{\partial Z_8}{\partial C_8}\Delta C_8$$

$$= \frac{(1 - \omega^2 R_8^2 C_8^2)\Delta R_8 - 2\omega^2 R_8^3 C_8 \Delta C_8}{(1 + \omega^2 R_8^2 C_8^2)^2}$$

$$- j\frac{2\omega R_8 C_8 \Delta R_8 + \omega R_8^2(1 - \omega^2 R_8^2 C_8^2)\Delta C_8}{(1 + \omega^2 R_8^2 C_8^2)^2}. \tag{60}$$

Substituting (44), (59), and (60) in (58) and solving for ΔR_8 and ΔC_8 gives

$$\Delta R_8 = \frac{R_8}{p_1^2 + 1}[\Delta p_1(\omega R_8 C_8 - p_1) + \Delta q_1(p_1\omega R_8 C_8 + 1)], \tag{61}$$

$$\Delta C_8 = \frac{1}{\omega R_8(1 + p_1^2)}[\Delta p_1(1 + p_1\omega R_8 C_8) + \Delta q_1(p_1 - \omega R_8 C_8)], \tag{62}$$

where p_1 is given by (29).

Equations (61) and (62) show that for the condition of p_1 and $\omega R_8 C_8$ small in comparison with unity, a capacitance change in either Z_1 or Z_2 is balanced principally by a change in R_7 or R_8 and a power-factor change in either Z_1 or Z_2 is balanced principally by a change in either C_7 or C_8. These equations also show that if C_7 and C_8 are kept as low as possible for measuring the difference in power factor between p_2 and p_1, the changes in R_8 or C_8 for changes in Δp_1 or Δq_1, respectively, may be made quite small.

If the capacitance balance of the bridge is obtained through a change in the capacitance of Z_2 (balancing through variable condensers Z_2 in Fig. 6) instead of through R_7 or R_8 then we may obtain equations giving Δq_2 and ΔC_8 for change in Δp_1 or Δq_1, using a method similar to the one for the derivation

of (61) and (62). This will give

$$\Delta q_2 = \frac{C_2 R_8}{C_1 R_7} \frac{\begin{matrix}(1+\omega^2 R_7^2 C_7^2)[\Delta q_1(1+p_1\omega R_8 C_8) \\ +\Delta p_1(\omega R_8 C_8 - p_1)]\end{matrix}}{\begin{matrix}[(1+\omega^2 R_7 R_8 C_7 C_8)(1+p_1\omega R_8 C_8) \\ -(\omega R_8 C_8 - \omega R_7 C_7)(\omega R_8 C_8 - p_1)]\end{matrix}}, \quad (63)$$

$$\Delta C_8 = \frac{\begin{matrix}(1+\omega^2 R_8^2 C_8^2)[\Delta p_1(1+\omega^2 R_7 R_8 C_7 C_8) \\ +\Delta q_1(\omega R_8 C_8 - \omega R_7 C_7)]\end{matrix}}{\omega R_8 \begin{matrix}[(1+\omega^2 R_7 R_8 C_7 C_8)(1+p_1\omega R_8 C_8) \\ -(\omega R_8 C_8 - \omega R_7 C_7)(\omega R_8 C_8 - p_1)]\end{matrix}}. \quad (64)$$

VII. CALCULATION OF MEASURED CAPACITANCE BY BRIDGE.

For the calculation of the dielectric constant of an oil sample or other capacitance measurements it is necessary to determine capacitances from the bridge balance equations in terms of a known capacitance. This may be obtained from (21) giving C_2 in terms of C_1. Equation (21) may be written in terms of m_7, m_8, p_{c7}, p_{c8}, R_7, R_8, and C_1, employing a method similar to the one used in obtaining p_2 in (32). This gives

$$C_2 = \frac{R_7}{R_8} C_1 \frac{[m_8^2 + (1+m_8 p_{c8})^2]}{\begin{matrix}[(1+m_7 p_{c7})(1+m_8 p_{c8}) + p_1(m_8(1+m_7 p_{c7}) \\ -m_7(1+m_8 p_{c8})) + m_7 m_8]\end{matrix}}. \quad (65)$$

In Table II there are given values of C_2 from (65), divided by C_2 from $\frac{R_7}{R_8} C_1$, for given values of m_7, m_8, p_{c7}, p_{c8}, and p_1.

TABLE II.

		$m_7 = m_8$	m_8				
p_1.	$p_{c7} = p_{c8}$.		10^{-4}.	10^{-3}.	10^{-2}.	10^{-1}.	5×10^{-1}.
0	0	10^{-4}	1	1	1	$1-1(10^{-6})$	$1-4(10^{-6})$
0	0	10^{-3}	1	1	$1-1(10^{-6})$	$1-10(10^{-6})$	$1-40(10^{-6})$
0	0	10^{-2}	$1-1(10^{-6})$	$1-10(10^{-6})$	$1-100(10^{-6})$	$1-989(10^{-6})$	$1-3984(10^{-6})$
0	10^{-2}	10^{-4}	1	1	1	$1-1(10^{-6})$	$1-4(10^{-6})$
0	10^{-2}	10^{-3}	$1-1(10^{-6})$	$1-1(10^{-6})$	$1-2(10^{-6})$	$1-11(10^{-6})$	$1-40(10^{-6})$
0	10^{-2}	10^{-2}	$1-101(10^{-6})$	$1-110(10^{-6})$	$1-200(10^{-6})$	$1-1086(10^{-6})$	$1-4031(10^{-6})$
10^{-4}	0	10^{-4}	1	1	1	$1-1(10^{-6})$	$1-4(10^{-6})$
10^{-4}	0	10^{-3}	1	1	$1-1(10^{-6})$	$1-10(10^{-6})$	$1-40(10^{-6})$
10^{-4}	0	10^{-2}	1	$1-9(10^{-6})$	$1-99(10^{-6})$	$1-988(10^{-6})$	$1-3983(10^{-6})$

Since C_2 in (65) is determined from a known capacitance C_1, it is necessary to have the condenser representing C_1 either calibrated from a standard capacitance or have the design of C_1 such that its capacitance may be determined to the desired accuracy from its dimensions. With a condenser of the design shown in Fig. 2 the relative dimensions may be made such that the capacitance may be calculated to sufficient accuracy from the dimensions.

VIII. SHIELD BALANCE.

In general it is desired to determine the effect of a shield unbalance on the power-factor and capacitance balance of the bridge. As shown in Part II the current through Z_9 or the potential across Z_9 (the detecting circuit) will be equal to zero, when Z_9 is connected across the bridge arms, for the condition of $Z_1Z_8 = Z_2Z_7$ and either $Z_1Z_6 = Z_3Z_7$ or $Z_2Z_6 = Z_3Z_8$. These conditions will obviously be satisfied (within the limits of bridge sensitivity) if with the detecting circuit across successively the bridge arms and then between either bridge arm and shield there is no measurable unbalance. Under this condition the potentials of points b_1, b_2, and s (Fig. 2) are equal and the bridge constants of power factor and capacitance may be determined directly from the relation $Z_1Z_8 = Z_2Z_7$.

A study of (1) will also show that the current through Z_9 or the potential across Z_9 will be equal to zero for the condition of $Z_1Z_8 = Z_2Z_7$ and $Z_5/Z_4 = Z_2/Z_1$. If these two conditions hold then we may again obtain the bridge constants of power factor and capacitance directly from the relation $Z_1Z_8 = Z_2Z_7$, even though in this case Z_1Z_6 is not equal to Z_3Z_7 and Z_2Z_6 is not equal to Z_3Z_8. The relation $Z_5/Z_4 = Z_2/Z_1$ may be obtained by having the detecting circuit Z_9 connected across the bridge arms and adjusting Z_4, Z_5, and Z_6 until a change in Z_6 gives no measurable unbalance of the bridge.

The effect of a shield unbalance on the bridge readings of power factor and capacitance may be determined from (1). If it is assumed that the bridge and shield are balanced, that is, (4), (5), and (6) hold, then a change in Z_6 from balance to a value $Z_6 + \Delta Z_6$ will require a change in, say, Z_1 from balance to a value $Z_1 + \Delta Z_1$ to give a current through or potential

across the detecting circuit equal to zero with the detecting circuit across the bridge arms. This means that in general if the shield is unbalanced the bridge circuit must also be unbalanced from the relation $Z_1Z_8 = Z_2Z_7$. Substituting in (1) for Z_6 and Z_1, $Z_6 + \Delta Z_6$ and $Z_1 + \Delta Z_1$, respectively, and solving for ΔZ_1, for I_9 equal to zero, gives

$$\Delta Z_1 = \frac{\Delta Z_6(Z_1Z_1Z_5 - Z_1Z_2Z_4)}{\begin{array}{l} Z_6(Z_2Z_4 + Z_4Z_5 + Z_3Z_5 + Z_3Z_4) + Z_3Z_4Z_5 \\ + \Delta Z_6(Z_2Z_4 + Z_4Z_5 + Z_3Z_4 + Z_3Z_5 - Z_1Z_5) \end{array}}. \quad (66)$$

Now ΔZ_1 and ΔZ_6 may be written

$$\Delta Z_1 = \Delta R_1 - j\Delta X_1, \quad (67)$$

$$\Delta Z_6 = \Delta R_6' - j\Delta X_6', \quad (68)$$

where $\Delta R_6'$ and $\Delta X_6'$ are the changes in the equivalent series resistance and reactance of Z_6. Solving (66), (67), and (68) for ΔR_1 and ΔX_1 gives

$$\Delta R_1 = \frac{\begin{array}{l} (\Delta R_6'h_1 + \Delta X_6'h_2)(\Delta R_6'h_5 + \Delta X_6'h_6 + h_3) \\ + (\Delta X_6'h_1 - \Delta R_6'h_2)(\Delta X_6'h_5 - \Delta R_6'h_6 - h_4) \end{array}}{(\Delta R_6'h_5 + \Delta X_6'h_6 + h_3)^2 + (\Delta X_6'h_5 - \Delta R_6'h_6 - h_4)^2} \quad (69)$$

and

$$\Delta X_1 = \frac{\begin{array}{l} (\Delta X_6'h_1 - \Delta R_6'h_2)(\Delta R_6'h_5 + \Delta X_6'h_6 + h_3) \\ - (\Delta R_6'h_1 + \Delta X_6'h_2)(\Delta X_6'h_5 - \Delta R_6'h_6 - h_4) \end{array}}{(\Delta R_6'h_5 + \Delta X_6'h_6 + h_3)^2 + (\Delta X_6'h_5 - \Delta R_6'h_6 - h_4)^2}, \quad (70)$$

where

$$h_1 = \frac{X_1^3}{k_2k_4k_5}\{p_1[k_2k_4(p_1p_5 - 1) - k_5(p_2p_4 - 1)] - [k_2k_4(p_1 + p_5) - k_5(p_2 + p_4)]\}, \quad (71)$$

$$h_2 = -\frac{X_1^3}{k_2k_4k_5}\{[k_2k_4(p_1p_5 - 1) - k_5(p_2p_4 - 1)] - p_1[k_2k_4(p_1 + p_5) - k_5(p_2 + p_4)]\}, \quad (72)$$

$$\begin{aligned} h_3 = {} & \frac{X_1^2}{k_2k_3k_4k_5}\{R_6'[k_3k_5(p_2p_4 - 1) + k_2k_5(p_3p_4 - 1) \\ & + k_2k_4(p_3p_5 - 1) + k_2k_3(p_4p_5 - 1)] \\ & - X_6'[k_3k_5(p_2 + p_4) + k_2k_5(p_3 + p_4) \\ & + k_2k_4(p_3 + p_5) + k_2k_3(p_4 + p_5)]\} \\ & + \frac{X_1^3}{k_3k_4k_5}[p_5(p_3p_4 - 1) - (p_3 + p_4)], \end{aligned} \quad (73)$$

$$h_4 = -\frac{X_1^2}{k_2k_3k_4k_5}\{X_6'[k_3k_5(p_2p_4 - 1) + k_2k_5(p_3p_4 - 1) + k_2k_4(p_3p_5 - 1) + k_2k_3(p_4p_5 - 1)] + R_6'[k_3k_5(p_2 + p_4) + k_2k_5(p_3 + p_4) + k_2k_4(p_3 + p_5) + k_2k_3(p_4 + p_5)]\} - \frac{X_1^3}{k_3k_4k_5}[p_5(p_3 + p_4) + (p_3p_4 - 1)], \quad (74)$$

$$h_5 = \frac{X_1^2}{k_2k_3k_4k_5}[k_3k_5(p_2p_4 - 1) + k_2k_5(p_3p_4 - 1) + k_2k_4(p_3p_5 - 1) + k_2k_3(p_4p_5 - 1)] - \frac{X_1^2}{k_5}(p_1p_5 - 1), \quad (75)$$

$$h_6 = -\frac{X_1^2}{k_2k_3k_4k_5}[k_3k_5(p_2 + p_4) + k_2k_5(p_3 + p_4) + k_2k_4(p_3 + p_5) + k_2k_3(p_4 + p_5)] + \frac{X_1^2}{k_5}(p_1 + p_5), \quad (76)$$

and, in general

$$k_n = \frac{X_1}{X_n} = \frac{C_n}{C_1}, \quad (77)$$

$$p_n = \omega R_n C_n. \quad (78)$$

A study of (69) and (70) will show that ΔR_1 and ΔX_1 will both be zero for, first, the condition of both $\Delta R_6'$ and $\Delta X_6'$ zero and, second, the condition of

$$\left(\frac{k_4}{k_5}\right)\left[\frac{(p_4p_5 + 1) + j(p_5 - p_4)}{p_4^2 + 1}\right] = \left[\frac{(p_1p_2 + 1) + j(p_2 - p_1)}{k_2(p_1^2 + 1)}\right] \quad (79)$$

for any values of either $\Delta R_6'$ or $\Delta X_6'$. However, it can also be shown from (69) that for a given $\Delta X_6'$ and certain bridge and shield constants ΔR_1 may be made equal to zero by a certain value of $\Delta R_6'$. In this case, however, a large value of ΔX_1 will result. This means that even if the shield is not balanced as far as X_6' is concerned, the power-factor unbalance

of the bridge may be made equal to zero through a compensating value of $\Delta R_6'$. This result may seem to indicate that it is not necessary in the shield balance to adjust X_6' for balance. However, it is necessary to have both $\Delta R_6'$ and $\Delta X_6'$ equal to zero, since (1) ΔR_1 in (69) cannot be made equal to zero by a definite value of $\Delta R_6'$ for any arbitrary value of $\Delta X_6'$ and certain bridge and shield constants, (2) the resulting large value of ΔX_1 due to adjustment of $\Delta R_6'$ to obtain the condition $\Delta R_1 = 0$ for a given $\Delta X_6'$ may introduce too great an error in the capacitance measurements, (3) in general a ΔX_1 will always remain; therefore when a vibration galvanometer is used the bridge unbalance will be so great as to prevent a satisfactory power-factor balance, and (4) even if an alternating-current galvanometer is used so that through the phase shifting of the galvanometer field it can be made insensitive to a given component of bridge unbalance, the large value of ΔX_1 will require an accuracy of phase-shifter setting which will be impracticable to obtain. The foregoing conclusions relative to an adjustment of $\Delta R_6'$ to give $\Delta R_1 = 0$ for a given $\Delta X_6'$ will apply similarly to the case of adjusting $\Delta X_6'$ to give $\Delta X_1' = 0$, for a given $\Delta R_6'$.

The values of $\Delta R_6'$ and $\Delta X_6'$ may be determined from Z_6 (Fig. 6) and the changes in R_6 and C_6 of Z_6. Thus if R_6 is changed to $R_6 + \Delta R_6$ and C_6 is changed to $C_6 + \Delta C_6$ where C_6 and R_6 refer to the balanced shield, then

$$Z_6 + \Delta Z_6 = \frac{(R_6 + \Delta R_6) - j\omega(R_6 + \Delta R_6)^2(C_6 + \Delta C_6)}{1 + \omega^2(R_6 + \Delta R_6)^2(C_6 + \Delta C_6)^2} \tag{80}$$

and

$$Z_6 = \frac{R_6 - j\omega R_6^2 C_6}{1 + \omega^2 R_6^2 C_6^2}. \tag{81}$$

Now $\Delta R_6'$ and $\Delta X_6'$ may be determined from (68), (80) and (81).

The accuracy with which the shield circuit must be balanced to prevent the bridge being unbalanced by a ΔR_1 or a ΔX_1 greater than a given amount may be determined approximately from (69) and (70). In general the effect of p_1, p_2, p_3, p_4 and p_5 on the required accuracy of the shield balance will be only a small percentage of what can be obtained through

R_6 and C_6. Then from (69) and (70), with p_1, p_2, p_3, p_4 and p_5 equal to zero, $k_2 = 1$, and for points not far removed from shield balance

$$\Delta R_1 = \frac{(\Delta R_6'X_6' - \Delta X_6'R_6')[X_1k_3(k_4 - k_5)(k_3k_5 + k_3 + k_4 + k_5)] + X_1^2\Delta R_6'k_3(k_4 - k_5)}{X_1^2 + 2X_1X_6'(k_3k_5 + k_3 + k_4 + k_5) + ((X_6')^2 + (R_6')^2)(k_3k_5 + k_3 + k_4 + k_5)^2}, \quad (82)$$

$$\Delta X_1 = \frac{(\Delta X_6'X_6' + \Delta R_6'R_6')[X_1k_3(k_4 - k_5)(k_3k_5 + k_3 + k_4 + k_5)] + X_1^2\Delta X_6'k_3(k_4 - k_5)}{X_1^2 + 2X_1X_6'(k_3k_5 + k_3 + k_4 + k_5) + ((X_6')^2 + (R_6')^2)(k_3k_5 + k_3 + k_4 + k_5)^2}. \quad (83)$$

In general, X_6' and R_6' are small in comparison with X_1 and therefore ΔR_1 from (82) may be approximately written for a given value of $\Delta R_6'$, and $\Delta X_6' = 0$,

$$\Delta R_1 \leqq \Delta R_6'k_3(k_4 - k_5). \quad (84)$$

Now, if it is desired to determine the change in $\Delta R_6'$ which will give a ΔR_1 corresponding to a change in p_1, or power factor of Z_1, of

$$\Delta p_1 = \Delta R_1\omega C_1 \quad (85)$$

we get from (84) and (85)

$$\Delta R_6' \leqq \frac{\Delta p_1}{\omega C_1k_3(k_4 - k_5)}. \quad (86)$$

Thus for the condition $\Delta p_1 = 10^{-6}$, $C_1 = 100(10^{-12})$ farads, $k_3 = 1$, $k_4 = 5$, $k_5 = 10$, and $\omega = 377$, $\Delta R_6'$ from (86) is approximately equal to 5.3 ohms. Obviously if C_4 and C_5 are made variable capacitances, they may be adjusted to be more nearly equal than as given by $k_4 = 5$ and $k_5 = 10$. Thus if we adjust $k_4 = 9$ and $k_5 = 10$, the value of $\Delta R_6'$ from (86) (for the other constants remaining the same as for the foregoing example) becomes approximately 26.5 ohms.

An inspection of (86) shows that the required accuracy of the shield balance in $\Delta R_6'$, for a given power-factor unbalance of the bridge and the shield balanced for $\Delta X_6'$, becomes greater for an increase in frequency, increase in capacitance C_1, increase in capacitance from high-tension to shield, C_3,

and for an increase in the difference between the capacitances between bridge and shield, C_4 and C_5.

Now if it is assumed that the shield is balanced for R_6, that is, $\Delta R_6' = 0$, but unbalanced by a $\Delta X_6'$, and again assuming that X_6' and R_6' are small in comparison with X_1 then from (83)

$$\Delta X_1 = \Delta X_6' k_3(k_4 - k_5). \tag{87}$$

Now for small changes in ΔX_1

$$\frac{\Delta X_1}{X_1} = -\frac{\Delta C_1}{C_1} = -\Delta q_1 \tag{88}$$

and for small changes in C_6

$$\Delta X_6' = \frac{\omega R_6^2 \Delta C_6}{1 + \omega^2 R_6^2 (C_6)^2}. \tag{89}$$

Then from (87), (88) and (89)

$$\Delta C_6 \cong -\frac{\Delta q_1(1 + \omega^2 R_6^2 C_6^2)}{\omega^2 R_6^2 k_3(k_4 - k_5) C_1}. \tag{90}$$

Thus with $\Delta q_1 = 10^{-6}$, that is, a capacitance unbalance of the bridge corresponding to $10^{-6}C_1$, $R_6 = 10^4$ ohms, $C_6 = 10{,}000(10^{-12})$ farads, $k_3 = 1$, $k_4 = 10$, $k_5 = 5$, and $\omega = 377$, ΔC_6 is equal to $140.9(10^{-12})$ farads. This gives the approximate change in ΔC_6 which will give a capacitance unbalance of the bridge equal to $10^{-6}C_1$. The required accuracy of adjusting C_6 for shield balance, for a given capacitance unbalance of the bridge and the shield balanced for $\Delta R_6'$, becomes greater as the frequency and R_6 are increased until $\omega^2 R_6^2 C_6^2$ becomes greater than unity, increases with an increase in C_1 and C_3, and increases with an increase in the difference between C_4 and C_5.

A study of (82) and (83) will show that for the condition of R_6 and X_6 small in comparison with X_1, k_3 and the difference between k_4 and k_5 relatively small, a change in the shield from balance by $\Delta R_6'$ (with $\Delta X_6' = 0$) will give principally a power-factor unbalance of the bridge and similarly a change in the shield from balance by $\Delta X_6'$ (with $\Delta R_6' = 0$) will give principally a capacitance unbalance of the bridge. However, if X_1 is not large in comparison with R_6 and X_6, and k_3 and

the difference between k_4 and k_5 are relatively large, then a change in the shield from balance of either $\Delta R_6'$ and $\Delta X_6'$ will give significant unbalances in the bridge of both power factor and capacitance. A study of (69) and (70) will also show that the effect of p_4 and p_5 will be to increase the power-factor unbalance of the bridge due to $\Delta X_6'$ even though $\Delta R_6'$ is equal to zero.

The effect of shield unbalance on the capacitance and power-factor balance of the bridge is shown in Figs. 8, 9, and 10. The capacitance and power-factor unbalance of the bridge is given in terms of Δp_1 (eq. 85) and Δq_1 (eq. 88). This means that if the bridge and shield are balanced and then the shield is changed to the values indicated, the tangent of the imperfection angle and capacitance of Z_1 would have to be changed by Δp_1 and Δq_1 for the bridge to be brought back to balance. In Fig. 8 there is shown the effect of changing R_6 (from balance), k_5, p_4 and p_5, for other given bridge constants. These curves show very clearly the desirability of having k_4 approximately equal to k_5. It is significant that Δp_1 is greater than Δq_1 for a given condition. In Figs. 9 and 10 there is shown the effect of changing R_6, p_4 and p_5, first, for the condition of $k_6 = 10$ and $k_7 = k_8 = 2$ and, second, for the condition of $k_6 = 40$ and $k_7 = k_8 = 2$. Thus in the case the shield is not balanced for phase, that is, independent of R_6' there will be a value of $\Delta X_6'$. It may be noted from Fig. 9 that for $p_4 = p_5 = 0$ and $k_6 = 40$ and $k_7 = k_8 = 2$, the value of Δp_1 is equal to zero for $R_6 = 10{,}004.6$ ohms. However, for this same condition from Fig. 10 the value of Δq_1 is $97(10^{-6})$. Also from Fig. 9 for $p_4 = p_5 = 0$ and $k_6 = 10$ and $k_7 = k_8 = 100$ the value of Δp_1 is equal to zero for $R_6 = 9980.4$ ohms. However, for this same condition from Fig. 10 the value of Δq_1 is $-231(10^{-6})$. These curves show that if the shield is not balanced for phase, for the bridge and shield constants indicated, the value of R_6 may be adjusted to give $\Delta p_1 = 0$, that is, the bridge is still balanced for power factor but there will remain a capacitance unbalance of the bridge. It is also shown that where the shield is not balanced for phase the effect of a power factor of Z_4 and Z_5 (p_4 and p_5) is to affect significantly the value of R_6 for $\Delta p_1 = 0$, whereas the change in Δq is very small. Likewise it can be shown

FIG. 8.

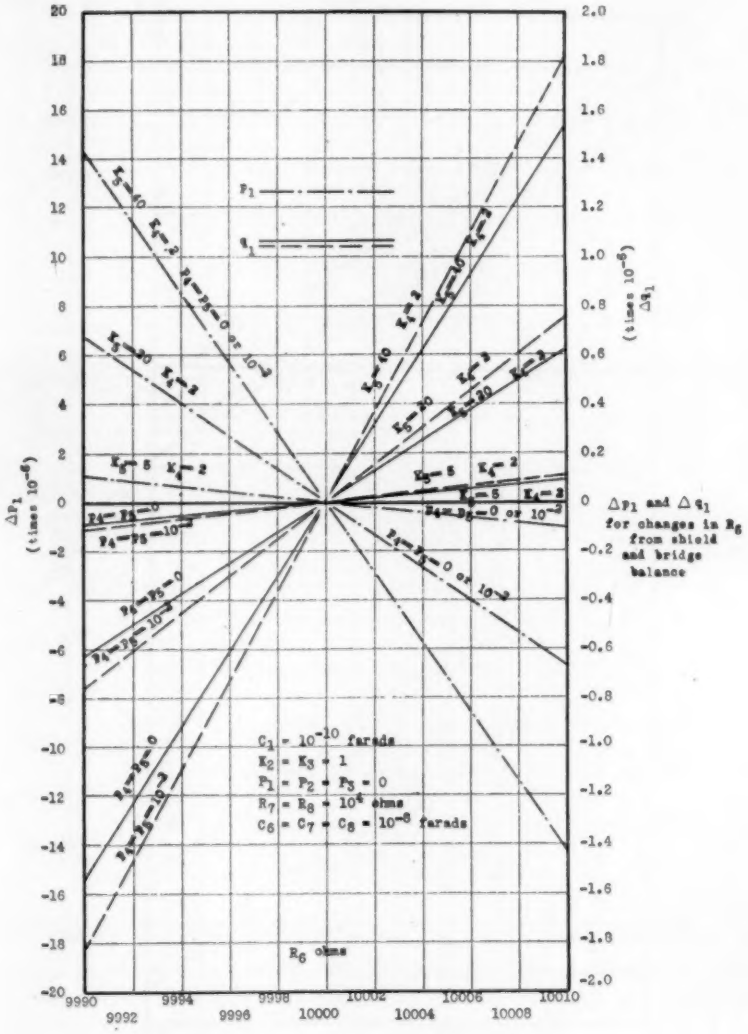
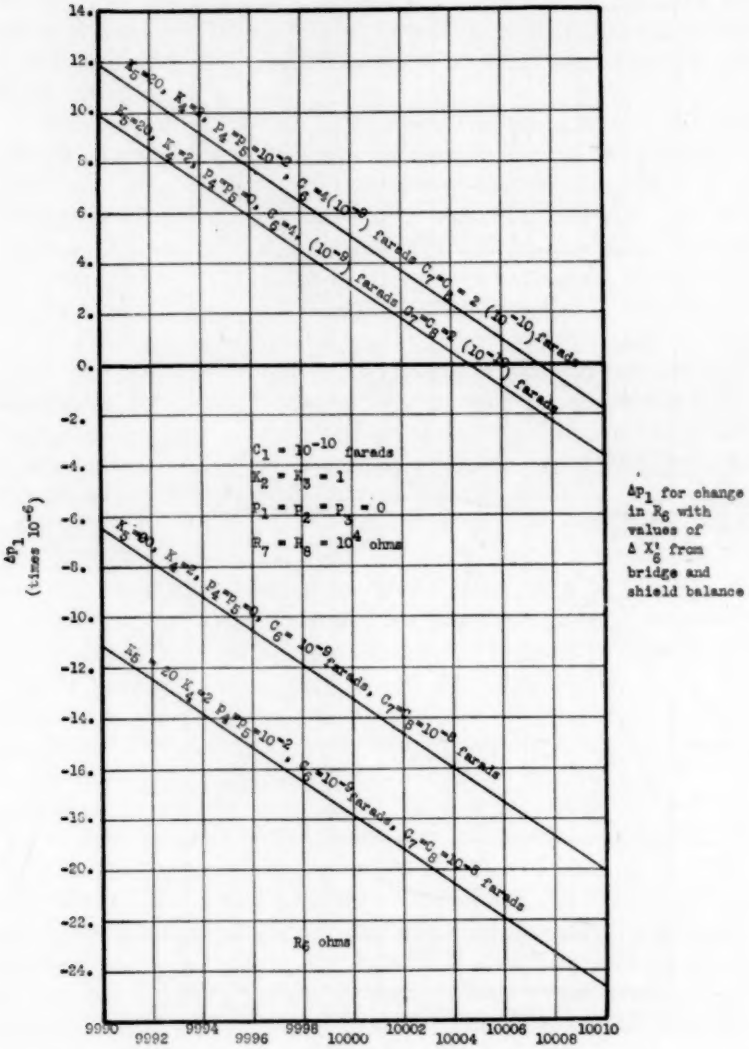

Δp_1 (times 10^{-6})
Δq_1 (times 10^{-6})
p_1
q_1
$K_5 = 10$ $K_4 = 2$ $P_4 = P_5 = 0$ or 10^{-2}
$K_5 = 20$ $K_4 = 2$
$K_5 = 5$ $K_4 = 2$
$P_4 = P_5 = 0$
$P_4 = P_5 = 10^{-2}$
$K_5 = 10$ $K_4 = 3$
$K_5 = 20$ $K_4 = 3$
$K_5 = 5$ $K_4 = 3$
$P_4 = P_5 = 0$ or 10^{-2}
Δp_1 and Δq_1 for changes in R_6 from shield and bridge balance
$C_1 = 10^{-10}$ farads
$K_2 = K_3 = 1$
$P_1 = P_2 = P_3 = 0$
$R_7 = R_8 = 10^4$ ohms
$C_6 = C_7 = C_8 = 10^{-8}$ farads
R_6 ohms
9990
9992
9994
9996
9998
10000
10002
10004
10006
10008
10010

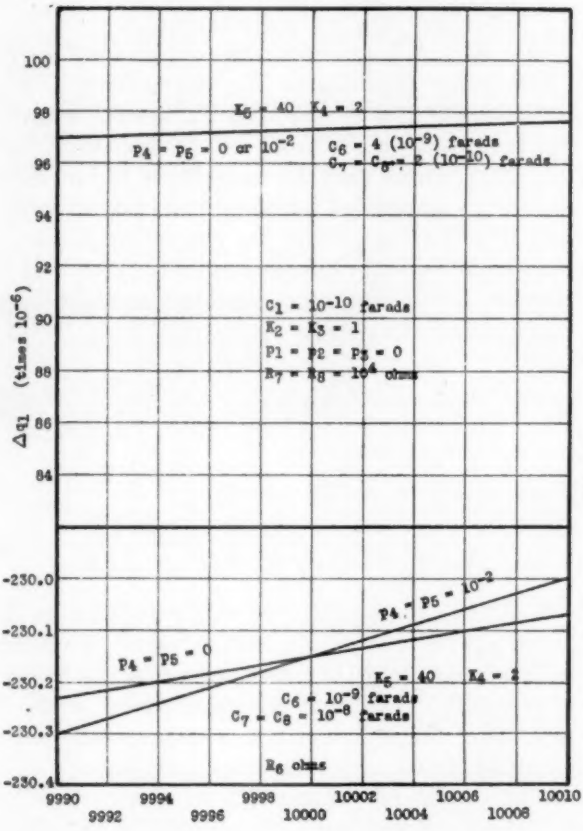
FIG. 9.



ΔP_1 for change in R_6 with values of $\Delta X'_6$ from bridge and shield balance

that if the shield is not balanced for R_6 (that is, there is a value of $\Delta R_6'$) there will be a value of $\Delta X_6'$ for the bridge and shield constants indicated in Figs. 8, 9 and 10 which will give $\Delta q_1 = 0$ but there will remain a value of Δp_1.

FIG. 10.



Δq_1 for change in R_6 with values of X_6' from bridge and shield balance

The shield circuit is balanced through the adjustment of $Z_6(R_6'$ and $\pm jX_6')$ with the detecting circuit connected between either bridge arm and shield. From (5) and (6) the value of Z_6 for balance is

$$Z_6 = \frac{Z_3 Z_7}{Z_1} = \frac{Z_3 Z_8}{Z_2}, \qquad (91)$$

from which

$$(R_6' - jX_6') = \frac{X_3}{X_1}\frac{R_7'(1 + p_1p_3) + X_7'(p_3 - p_1)}{p_1^2 + 1} - j\frac{X_3}{X_1}\frac{R_7'(p_1 - p_3) + X_7'(1 - p_1p_3)}{p_1^2 + 1}, \quad (92)$$

where R_6', R_7' and X_6', X_7' represent the equivalent series resistance and reactance of Z_6 and Z_7, respectively. For relatively low values of p_1 and p_3, X_6' and R_6' are determined principally from the values of X_7' and R_7', respectively, and the ratio of X_3 to X_1. There may be cases (due to the large inherent capacitance to ground of the shield circuit) in which a value of $+ j\Delta X_6'$ will be required for the shield balance. This would mean that an inductance would be required in place of C_6. For the condition of the bridge and shield in balance the shield voltage V_s will be

$$V_s = \frac{EZ_6}{Z_3 + Z_6}, \quad (93)$$

from which

$$V_s = \frac{E\{[R_6'(X_3p_3 + R_6') + X_6'(X_3 + X_6')] - j[X_6'(X_3p_3 + R_6') - R_6'(X_3 + X_6')]\}}{X_3^2(p_3^2 + 1) + 2X_3(X_6' + p_3R_6') + (R_6')^2 + (X_6')^2}. \quad (94)$$

An inspection of (94) shows that for the condition of R_6' and X_6' small in comparison with X_3, and p_3 small, a change in X_6' gives principally a voltage in phase with E and a change in R_6' gives principally a voltage in quadrature with E. Therefore for adjusting X_6' we may use a component of the applied bridge voltage.

The approximate value of voltage V_{s1} in phase with the applied bridge voltage E which must be applied to the shield circuit for shield balancing, may be determined from the difference between the shield voltage V_s and the voltage of either bridge arm. Thus

$$V_{s1} \cong \frac{EZ_7}{Z_1 + Z_7} - \frac{EZ_6}{Z_3 + Z_6} \quad (95)$$

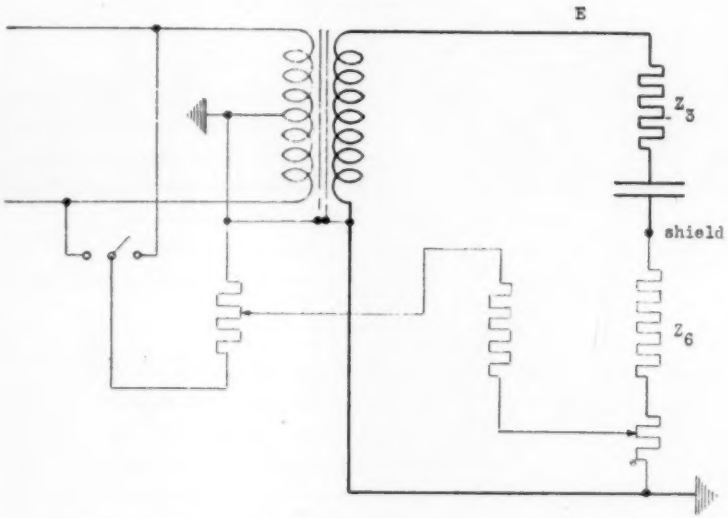
and the imaginary component of (95) must be balanced by R_6.

Equating V_{s1} to the real components of (95) gives

$$V_{s1} \cong \frac{E(R_7'(X_1p_1 + R_7') + X_7'(X_1 + X_7'))}{X_1^2(p_1^2 + 1) + 2X_1(X_7' + p_1R_7') + (R_7')^2 + (X_7')^2} - \frac{E(R_6'(X_3p_3 + R_6') + X_6'(X_3 + X_6'))}{X_3^2(p_3^2 + 1) + 2X_3(X_6' + p_3R_6') + (R_6')^2 + X_6)^2}. \quad (96)$$

The diagram of connections giving the method of obtaining a component of the applied voltage for the shield circuit is shown in Fig. 11.

FIG. 11.



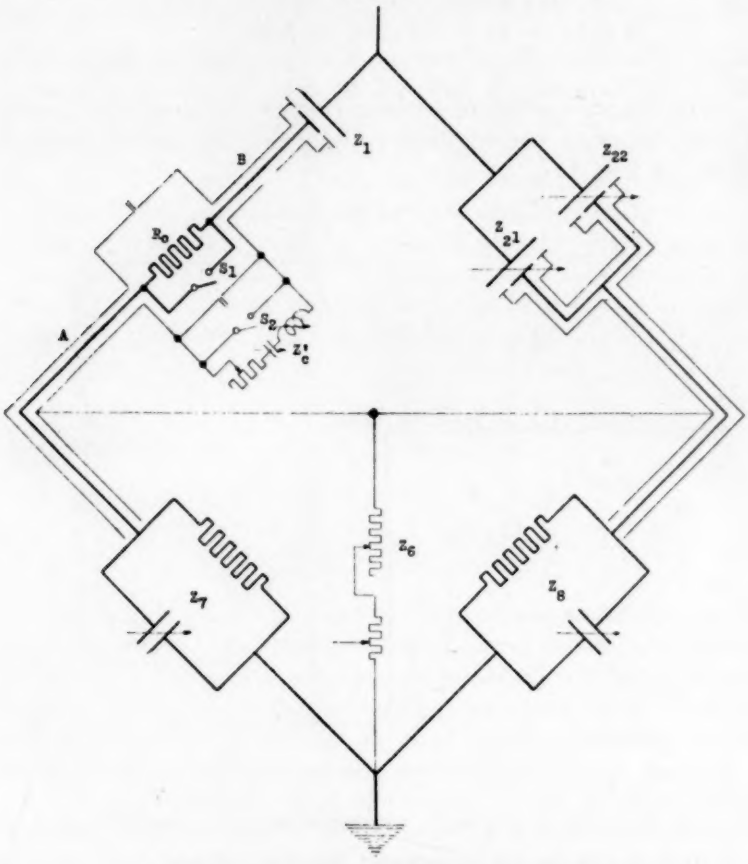
Connections for introducing into shield circuit component of applied bridge voltage.

IX. METHOD OF CHECKING MEASURED BRIDGE POWER FACTOR.

The power factor measured by the bridge may be conveniently and accurately checked by connecting a resistance R_C in series with the fixed impedance Z_1 as shown in Fig. 12. This resistance R_C should be connected in series with the bridge section of the impedance Z_1 and not in series with the high-tension section of Z_1. Connecting R_C in the high-tension side of Z_1 requires an evaluation of the capacitance

from high-tension to shield and to bridge, in addition to greater shielding difficulties, whereas if R_C is connected in the bridge side of Z_1 it is only necessary to know the capacitance from high-tension to bridge.

FIG. 12.



Circuit for checking measured bridge power factor.

When R_C is connected in series with the bridge section of Z_1 there will be a voltage drop across R_C (with shields balanced) equal to R_C times the current flowing through the bridge section of Z_1. Obviously therefore for the shields A and B in Fig. 12 on both sides of R_C to be balanced with respect

to the bridge on corresponding sides of R_C requires an impedance Z_C' connected across the shields A and B. The impedance drop across Z_C' should be adjusted to be equal to the resistance drop across R_C. The resistance component of Z_C' will be approximately equal to the ratio of the capacitance of Z_1 to shield to the capacitance of Z_1 to bridge times the resistance R_C. Due to the probable difference in phase of the currents flowing from high-tension to bridge and to shield in Z_1 and the capacitance of the shield to ground on the Z_1 side of R_C an accurate adjustment of the potentials of the shield A and B may also require either an inductance or capacitance component in Z_C'.

The bridge circuit is balanced in the usual way by connecting the detecting circuit across the bridge arms b_1 and b_2 in Fig. 6. The shield circuit on each side of R_C is balanced by connecting the detecting circuit between bridge and shield on corresponding sides of R_C and adjusting Z_C' for shield balance on the impedance Z_1 side of R_C and the usual shield balancing Z_6 for the shield balance on the other side of R_C.

With the resistance R_C shorted by means of switch S_1 and Z_C' shorted by means of switch S_2 balance of bridge and shield will give from (32) ($R_7 = R_8 = R$ and arms direct)

$$p_1 - p_2 = \omega R(C_{8d1} - C_{7d1}). \tag{97}$$

Now if R_C is connected in the bridge circuit by opening S_1 and the bridge and shields on both sides of R_C are balanced, rebalance of the bridge will require an adjustment of C_8 or C_7 and probably a very small change in capacitance of the vernier variable condenser Z_{22}. Assuming C_8 is adjusted from C_{8d1} to C_{8d2} for bridge balance (and the probable change in Z_{22} will be entirely negligible) then the bridge measures

$$p_1 + \Delta p_1 - p_2 = \omega R(C_{8d2} - C_{7d1}), \tag{98}$$

where

$$\Delta p_1 = \omega R_C C_1$$

and C_1 is the capacitance in farads of Z_1. Subtracting (97) from (98) gives

$$\Delta p_1 = \omega R_C C_1 = \omega R(C_{8d2} - C_{8d1}), \tag{99}$$

from which differences in capacitance of C_8 for different

settings may be determined directly in terms of a given Δp_1 or change in power factor.

With the resistance R_C as shown in Fig. 12 perfect shielding is not provided by the shield circuit. It is therefore necessary to investigate the effect of this imperfect shielding on the accuracy of the bridge power-factor and capacitance balances. This may be done through (69) and (70) or (82) and (83). If the bridge and shield circuits on both sides of R_C are balanced then Z_4 in (69) and (70) or (82) and (83) will approximate the capacitance of the resistance R_C to shield and Z_5 will be equal to infinity. Such a study will show that if the capacitance from R_C to shield is kept relatively small in comparison with the capacitance of Z_1 this effect will be entirely negligible in the 10^{-6} range for values of R_C representing power factors in the range of 10^{-5}. Furthermore the effect of the imperfect shielding of R_C will be greatly decreased by the location of R_C symmetrical with respect to shields A and B.

It is of course possible to design R_C and the shield impedance in such a way that nearly perfect shielding of R_C may be obtained. This in general will not be required since it is unnecessary to check for values of R_C representing large power factors. Furthermore it would be simpler for checking large power factors to obtain differences in capacitance of a correctly designed condenser as shown in Part III.

X. MEASUREMENT OF THE POWER FACTOR OF AN OIL SAMPLE.

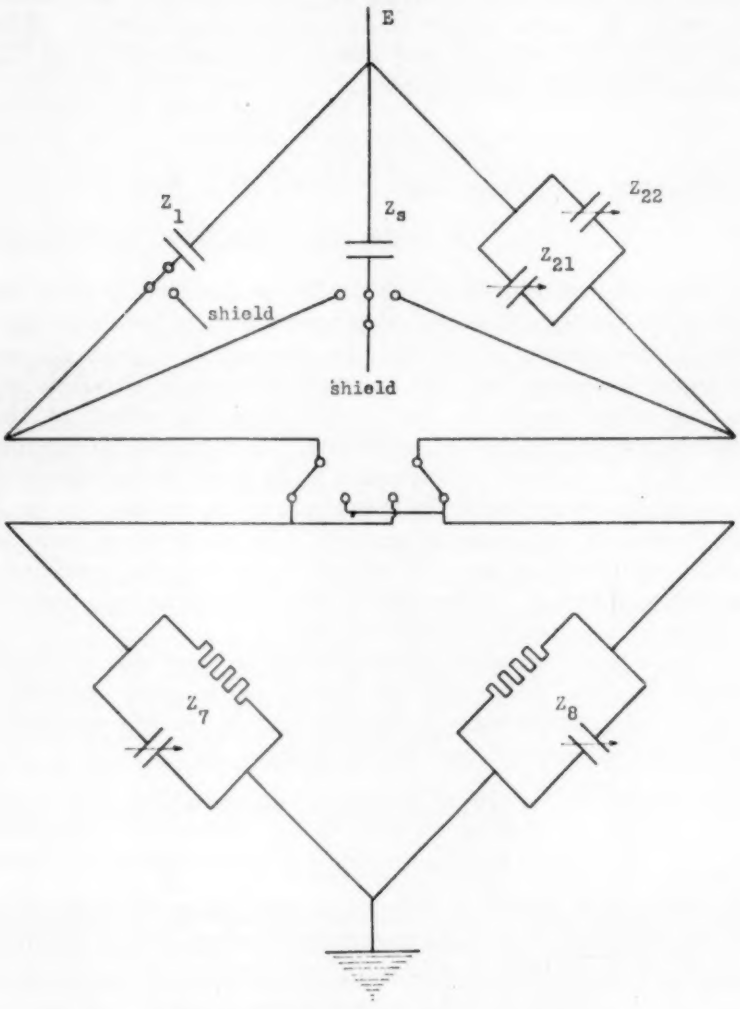
Consider the bridge diagram of Fig. (13) in which Z_1 is a fixed impedance, Z_{21} and Z_{22} are variable impedances and Z_s is the oil sample.

If first the sample is connected to the shield, then balance of bridge and shield with arms connected direct (as shown in Fig. (13)) from (32) gives

$$p_2 - p_1 = \omega R_7 C_{7d1} - \omega R_8 C_{8d1}, \qquad (100)$$

where p_1 is the tangent of the imperfection angle of the impedance represented by Z_1, and p_2 is the tangent of the imperfection angle of the combination represented by Z_{21} and Z_{22}. Now if the oil sample Z_s is connected into the bridge circuit in place of Z_1, and Z_1 is connected to shield, and as-

FIG. 13.



Connections for oil sample power-factor measurements.

suming $C_s = C_1$, the capacitances of Z_s and Z_1, respectively, rebalance of the bridge may be obtained through adjusting either C_7 or C_8 (assuming p_s, the tangent of the imperfection angle of Z_s is different than p_1) and the variable capacitances C_{21} or C_{22}, the capacitances of Z_{21} and Z_{22}, respectively. With the shield also balanced and assuming C_8 adjusted from C_{8d1} to C_{8d2} for bridge balance, we get

$$p_2 - p_s = \omega R_7 C_{7d1} - \omega R_8 C_{8d2}. \tag{101}$$

Subtracting (101) from (100) gives for $R_7 = R_8 = R$

$$p_s - p_1 = \omega R(C_{8d2} - C_{8d1}). \tag{102}$$

Thus the measured power factor of the bridge gives the difference between p_s of the oil sample and p_1 of the impedance which the sample replaced. In general if C_{21} or C_{22} are adjusted between the two balances then p_2 in (100) is not exactly equal to p_2 in (101). However, the effect of this change will in general be negligible. It is shown from (102) that if it is desired to measure p_s to a given accuracy, p_1 must either be negligible in this range or the value of p_1 must be known to the same accuracy. The value of p_1 may be made negligible in the 10^{-6} range by having the measuring section surfaces of C_1 free from oxides and clean and under a relatively low gradient (see Part XI).

The power factor of the oil sample p_s may also be determined by first balancing the bridge with arms connected direct and reverse with Z_1 and Z_{21} and Z_{22} in the respective high-tension arms. Thus for $R_7 = R_8 = R$.

$$p_2 - p_1 = \omega R(C_{7d1} - C_{8d1}) \quad \text{(arms direct)}, \tag{103}$$

$$p_2 - p_1 = \omega R(C_{8d2} - C_{7d1}) \quad \text{(arms reverse)}, \tag{104}$$

in which it is assumed that C_7 is not changed in the two balances and C_8 is changed (assuming a value of $p_2 - p_1$) from C_{8d1} to C_{8d2}. Adding (103) and (104) gives

$$(p_2 - p_1) = \omega R \frac{(C_{8d2} - C_{8d1})}{2} = A_1. \tag{105}$$

Now if the oil sample, Z_s, is connected in place of Z_1 and the

bridge balanced with arms connected direct and reverse then

$$p_2 - p_s = \omega R(C_{7d3} - C_{8d3}) \quad \text{(arms direct)}, \qquad (106)$$

$$p_2 - p_s = \omega R(C_{8d4} - C_{7d3}) \quad \text{(arms reverse)}, \qquad (107)$$

in which again C_7 is not changed in the two balances and C_8 is adjusted from C_{8d3} to C_{8d4} (assuming a value of $p_2 - p_s$). Adding (106) and (107) gives

$$p_2 - p_s = \omega R\frac{(C_{8d4} - C_{8d3})}{2} = A_2. \qquad (108)$$

Subtracting (108) from (105) gives

$$p_s - p_1 = A_1 - A_2, \qquad (109)$$

from which again p_s may be determined provided p_1 is known or is negligible in the desired accuracy range of p_s. Of course if p_2 of the variable capacitances is known or is negligible in the desired power-factor range then p_s may be obtained directly from (108) or (101). From a construction and cost point of view it is less desirable to have a variable capacitance with a relatively low gradient than a fixed capacitance. Furthermore it is considerably easier to disassemble and clean a fixed capacitance than a variable capacitance.

The power factor of the oil sample may also be determined by connecting the sample into the bridge circuit in parallel with Z_{21} and Z_{22}. Thus if first the sample is connected to shield and Z_1 is balanced against Z_{21} and Z_{22} with the arms connected direct and reverse, $p_2 - p_1$ will be given by (105). Now if the sample is connected in parallel with Z_{21} and Z_{22}, the bridge may again be balanced against Z_1 with the arms connected direct and reverse and will give

$$p_{2s}' - p_1 = \omega R(C_{8d6} - C_{8d5}) = A_3, \qquad (110)$$

where p_{2s}' is the tangent of the imperfection angle of the combination Z_s, Z_{21}' and Z_{22}', Z_{21}' and Z_{22}' representing the impedances of Z_{21} and Z_{22} after balance in parallel with Z_s against Z_1. Balance of the bridge with the sample connected in parallel with Z_{21} and Z_{22} will require an adjustment of Z_{21} and C_{22} from an initial value of

$$C_{21} + C_{22} \leqq C_1 \qquad (111)$$

to

$$C_{21}' + C_{22}' + C_s \leqq C_1. \tag{112}$$

Subtracting (105) from (110) gives

$$p_{2s}' - p_2 = A_3 - A_1. \tag{113}$$

Thus in this case the power factor of $Z_1(p_1)$ is eliminated but now the values of p_2 for Z_{21} and Z_{22} (for $C_{21} + C_{22} \leqq C_1$) in the first balances when Z_{21} and Z_{22} are balanced against Z_1, and of p_2' for Z_{21}' and Z_{22}' (for $C_{21}' + C_{22}' \leqq C_1 - C_s$) in the second balances when Z_{21}', Z_{22}' and Z_s are balanced against Z_1 must be known. Also it is necessary to evaluate p_s from p_{2s}'.

When two impedances Z_a and Z_b are connected in parallel having capacitances C_a and C_b, respectively, and tangents of imperfection angle p_a and p_b, respectively, the value of p_{ab} of the combination is given by

$$p_{ab} = \frac{p_a C_a(p_b^2 + 1) + p_b C_b(p_a^2 + 1)}{C_a(p_b^2 + 1) + C_b(p_a^2 + 1)}. \tag{114}$$

Now if p_{2s}' is calculated from (113) knowing p_2, A_3 and A_1, p_s may be obtained from (114) through p_{2s}', p_2', $C_{21}' + C_{22}'$ and C_s. This gives

$$p_s = \frac{\begin{array}{r} C_s((p_2')^2 + 1) \pm [C_s^2((p_2')^2 + 1)^2 - 4C_s(C_{21}' \\ + C_{22}')p_{2s}'((p_2')^2 + 1)(p_{2s}' - (p_2') \\ - 4(C_{21}' + C_{22}')(p_{2s}' - p_2')^2]^{1/2} \end{array}}{2(C_{21}' + C_{22}')(p_{2s}' - p^{2\prime})}. \tag{115}$$

When p_a and p_b are very small in comparison with unity p_{ab} in (115) reduces to

$$p_{ab} \leqq p_a \frac{C_a}{C_a + C_b} + p_b \frac{C_b}{C_a + C_b}, \tag{116}$$

and calculating p_s again from (116) instead of (114) gives

$$p_s \leqq p_{2s}'\left(\frac{C_{21}' + C_{22}' + C_s}{C_s}\right) - p_2'\left(\frac{C_{21}' + C_{22}'}{C_s}\right). \tag{117}$$

The difference between p_s as obtained from (117) and (115) will in general depend upon the relative values of p_{2s}', p_2', $C_{21}' + C_{22}'$ and C_s. The magnitude of this difference

for given ratios of C_{21}' and $C_{22}' + C_s$ to C_s, and values of p_{2s}' and p_2' is given in Table III.

TABLE III.

$\frac{C_{21}'+C_{22}'+C_s}{C_s}$.	p_{2s}'.	p_2'.	p_s from (115).	p_s from (117).
3	10^{-4}	0	$4(10^{-4})$	$4(10^{-4})$
3	10^{-2}	0	$4(10^{-2})+48(10^{-6})$	$4(10^{-2})$
3	$5(10^{-2})$	0	$2(10^{-1})+6389(10^{-6})$	$2(10^{-1})$
3	10^{-4}	10^{-4}	$4(10^{-4})-3(10^{-4})$	$4(10^{-4})-3(10^{-4})$
3	10^{-2}	10^{-4}	$4(10^{-2})-257(10^{-6})$	$4(10^{-2})-3(10^{-4})$
3	$5(10^{-2})$	10^{-4}	$2(10^{-1})+6059(10^{-6})$	$2(10^{-1})-3(10^{-4})$
1/3	10^{-4}	0	$4/3(10^{-4})$	$4/3(10^{-4})$
1/3	10^{-2}	0	$4/3(10^{-2})+1(10^{-6})$	$4/3(10^{-2})$
1/3	$5(10^{-2})$	0	$2/3(10^{-1})+74(10^{-6})$	$2/3(10^{-1})$
1/3	10^{-4}	10^{-4}	$4/3(10^{-4})$	$4/3(10^{-4})-1/3(10^{-4})$
1/3	10^{-2}	10^{-4}	$4/3(10^{-2})-34(10^{-6})$	$4/3(10^{-2})-1/3(10^{-4})$
1/3	$5(10^{-2})$	10^{-4}	$2/3(10^{-1})+41(10^{-6})$	$2/3(10^{-2})-1/3(10^{-4})$

Table III shows that there may be quite large differences between p_s as given by (117) and (115). These results show that in general for the power-factor measurements of small oil samples it is more satisfactory to substitute the oil sample for a fixed condenser of approximately the same capacitance than by placing the oil sample in parallel with a variable capacitance. The main advantage of the latter method is the ease with which a capacitance balance may be obtained with different capacitances for different oil samples and the change in dielectric constant with temperature, etc.

With the oil sample substituted for a fixed condenser (Z_1 in Fig. 13) it is desirable to have the capacitance of Z_1 variable through fixed steps to take into account changes in capacitance of the sample. This may be easily accomplished through the design of a condenser similar to that shown in Fig. 2 but with the measuring section divided into sections. The lead from each of these sections may be taken out of the condenser through the shield supporting shaft (Fig. 2) and into a shielded switch box. This will permit any section of the condenser (except of course the end shield sections) to be connected either to bridge or to shield and thereby give approximately a capacitance equal to that of the oil cell which replaces the condenser.

XI. EVALUATION OF POWER FACTOR OF AIR CONDENSERS.

Consider the diagrammatic plan of the bridge circuit in Fig. 14. In this diagram Z_{11}, Z_{12}, Z_{13}, and Z_{14} represent fixed condensers of identical type of construction (Fig. 2) with the measuring and high-tension sections made from the same kind of metal, and of approximately equal capacitances but with different spacings. The measuring surfaces of each of these condensers are initially similarly treated to remove the surface oxides (when these may be removed) and then cleaned with carbon tetrachloride. The object is to obtain similar surfaces in each of the capacitances.

The connections as shown in Fig. 14 are such that any of the fixed capacitances may be connected either to bridge or shield and the low-tension measuring arms may be connected direct or reverse. With Z_{11}, Z_{12}, Z_{13} and Z_{14} successively balanced against Z_{21} and Z_{22}, obviously we can obtain the following differences in power factors, expressed as B_1, B_2, B_3 and B_4.

$$p_{11} - p_2 = B_1, \tag{118}$$

$$p_{12} - p_2 = B_2, \tag{119}$$

$$p_{13} - p_2 = B_3, \tag{120}$$

$$p_{14} - p_2 = B_4, \tag{121}$$

where p_2 represents the tangent of the imperfection angle of impedances Z_{21} and Z_{22} and p_{12}, p_{13}, p_{14} and p_{14} represent the tangents of the imperfection angles of Z_{11}, Z_{12}, Z_{13} and Z_{14}, respectively. From (118), (119), (120) and (121) can be obtained the difference in power factors of any of the fixed capacitances.

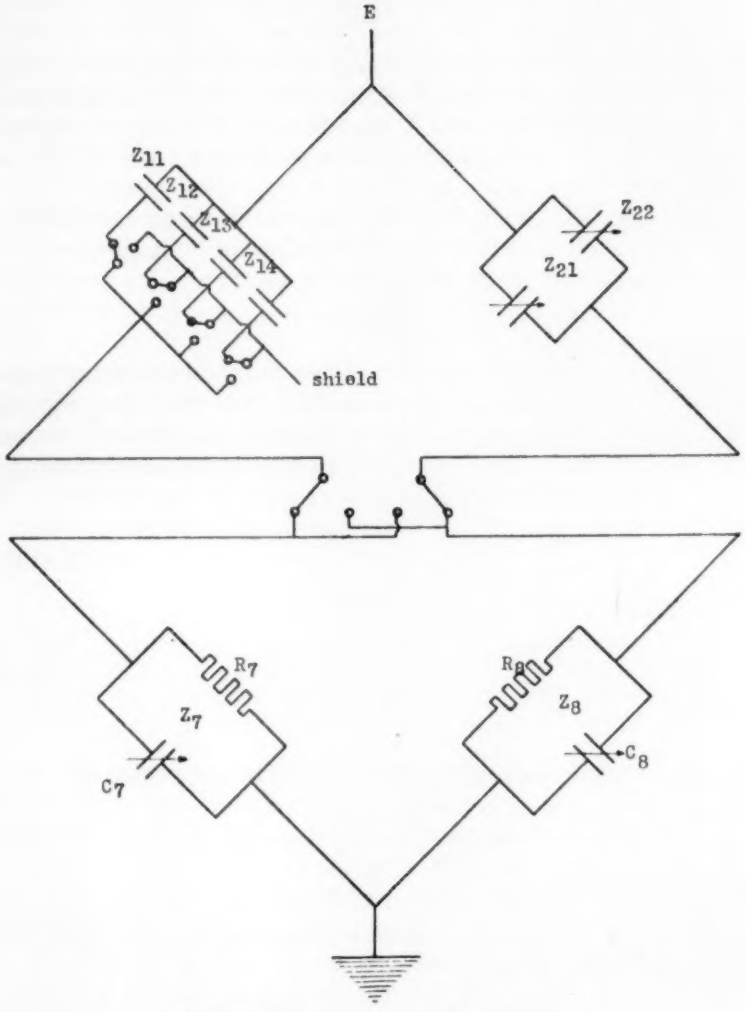
The value of p_{11} of the fixed impedance Z_{11} from (23) is

$$p_{11} = \omega R_{11} C_{11}, \tag{122}$$

where C_{11} is the capacitance of Z_{11} in farads, R_{11} is the equivalent series resistance and ω equals 2π times the frequency in cycles per second. The equivalent series resistance R_{11} may be expressed as

$$R_{11} = \frac{W_{11}}{\omega^2 E^2 (C_{11})^2} + \frac{W_{11}(p_{11})^2}{\omega^2 E^2 (C_{11})^2}. \tag{123}$$

FIG. 14.



Circuit for evaluation of power-factor of condensers.

In general p_{11}^2 will be quite small in comparison with unity and therefore from (122) and (123)

$$p_{11} = \frac{W_{11}}{\omega E^2 C_{11}}, \tag{124}$$

where W_{11} is the loss in watts in Z_{11} and E is the applied voltage to Z_{11} in volts. In (123) if R_{11} from the first term represents a p_{11} corresponding to 10^{-4} then the second term represents a value of R_{11} corresponding to a p_{11} of 10^{-8} which will be negligible.

The loss W_{11} of Z_{11} may be assumed to be proportional to the surface area and to some power of the voltage gradient. Then the loss W_{11} may be expressed as

$$W_{11} = K_A L_{11}(r_{11a}E_{11a}^{x} + r_{11b}E_{11b}^{x}), \tag{125}$$

in which K_A is a constant, r_{11a} and r_{11b} are the inner and outer radii, respectively, of Z_{11}, E_{11a} and E_{11b} are the voltage gradients on the surface areas corresponding to r_{11a} and r_{11b}, respectively, and L_{11} is the length of the measuring section of Z_{11}. The voltage gradients E_{11a} and E_{11b} may be expressed as

$$E_{11a} = \frac{E}{r_{11a} ln \frac{r_{11b}}{r_{11a}}}, \tag{126}$$

$$E_{11b} = \frac{E}{r_{11b} ln \frac{r_{11b}}{r_{11a}}}. \tag{127}$$

Also the capacitance C_{11} of Z_{11} may be expressed as

$$C_{11} = K_B \frac{L_{11}}{ln \frac{r_{11b}}{r_{11a}}} \text{ farads}, \tag{128}$$

where K_B is a constant. Substituting (125), (126), (127) and (128) and (124) gives

$$p_{11} = \frac{K_A}{K_B} \frac{E^{x-2}}{\omega} (r_{11a} + r_{11b})^{1-x} \left(ln \frac{r_{11b}}{r_{11a}} \right)^{1-x}. \tag{129}$$

By (129) and analogous equations for p_{12}, p_{13} and p_{14} and the differences between any values of p_{11}, p_{12}, p_{13} and p_{14}

from (118), (119), (120) and (121), the values of p_{11}, p_{12}, p_{13} and p_{14} may be evaluated and also the value of any p as a function of applied voltage.

Figure 15 shows the results obtained from a series of tests made on a group of four condensers, the high-voltage, shield and measuring surfaces of which were made from standard brass tubing. These condensers each had a capacitance of approximately 86.8×10^{-12} farads and the spacings of Z_{11}, Z_{12}, Z_{13} and Z_{14} were approximately 1/16″, 3/16″, 7/16″ and 15/16″, respectively, and the external diameter of the bridge cylinder was approximately 2″ in each condenser. The tests were made with an applied bridge voltage of 800 volts at a frequency of 55 cycles per second. The values plotted are the differences between p_{11}, p_{12}, p_{14} and p_{13}. The measuring surfaces of these condensers were initially polished with a fine emery cloth to remove surface oxides as completely as possible and then cleaned with carbon tetrachloride. Following this the condensers were assembled without touching the measuring surfaces and differences between absolute power factors obtained from bridge measurements. An evaluation of the absolute power factors as described previously was made and found to be

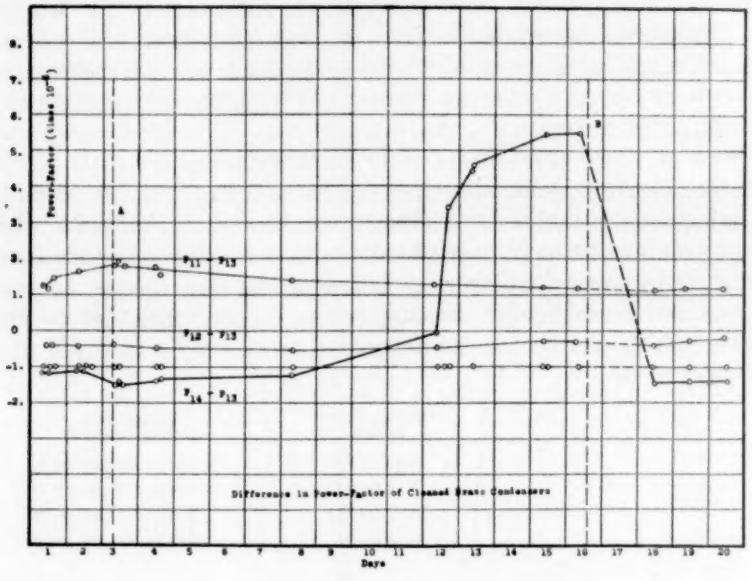
	Absolute Power Factor.	Approximate Average Gradient.
Z_{11}	$3.1(10^{-6})$	13,000 volts per inch
Z_{12}	$1.0(10^{-6})$	4,300 volts per inch
Z_{13}	$0.4(10^{-6})$	1,800 volts per inch
Z_{14}	$0.2(10^{-6})$	900 volts per inch

It was also found that the loss in the condensers was proportional to the square of the applied voltage and thus the absolute power factor of a given condenser would be independent of the applied voltage. This conclusion was also checked through measurements made at different applied voltages.

At time A in Fig. 15 the measuring surfaces of Z_{14} were dampened by means of a wet cloth. It is interesting to note that immediately following dampening of the surfaces, the value of $(p_{14} - p_{13})$ decreased by approximately the value of p_{14}. Following this the absolute power factor of Z_{14} increased at first quite slowly and finally quite rapidly and then ap-

proached an approximately constant value of $7(10^{-6})$. This condenser was then disassembled and it was noted that the measuring surfaces had become slightly oxidized. Furthermore after this condenser was cleaned (time *B* Fig. 15) similarly to the initial cleaning the absolute power factor of the condenser returned to approximately its initial value. This series of tests was performed a second time with a

FIG. 15.



different set of brass tubing and the results were substantially the same.

While these power factors are relatively low, it is evident that even with the brass surfaces cleaned as effectively as possible the power factor of a brass condenser will be significant in the 10^{-6} range with a gradient of the order of 5000 volts per inch. A series of tests was also made on brass tubing as received, which in general has a substantial amount of surface oxides, and it was found that the power factor for a gradient of 10,000 volts per inch may be substantially greater than 10^{-4}. In this case cleaning the surfaces with carbon

tetrachloride had only a small effect on the power factor. It was also found that the power factor of the condenser varied in the range of the square of the voltage gradient. However, due to the difficulty in obtaining surfaces with the same amount of surface oxides it was not possible to obtain this value to a high degree of accuracy.

A number of tests were also made with the measuring surfaces of the condensers made of aluminum and also of chromium plated brass. It was found in these tests, that in general, with the surfaces cleaned as effectively as possible, the power factor was greater than with brass surfaces and similar voltage gradients.

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Of Insects and Seeds (U. S. Department of Agriculture Clip Sheet No. 825).—Just to what extent white clover depends on insects for the cross pollination necessary to seed production has been recently demonstrated by entomologists of the U. S. Department of Agriculture, Louisiana. Three plots of white clover at each of three separate places were chosen for the experiment. One plot in each locality was enclosed with wire cloth having 60 meshes to the inch, calculated to keep out all insects; one with 6-mesh screening which prevented the entrance of honeybees and other insects of the same size or larger; and one with chicken wire, which permitted free passage to all insects, but protected the clover against grazing animals.

In the two years of the investigation, at all three locations, only 5 seeds were obtained from the three 60-mesh cages; 6599 seeds were obtained from the 6-mesh cages, and 56,365 seeds were obtained from the unscreened cages.

C.

Charting Rainfall Intensities.—To successfully design a municipal storm sewer system large enough to take care of the storm water safely and within the appropriations available, engineers must know how much rain is ever likely to fall in 5 minutes, in 15 minutes and in an hour or more. They also need to know how often these sudden downpours are likely to occur. Engineers planning terracing systems on farmlands, farm drainage systems for surface water, and culvert waterway openings for small watersheds must also have this information for best results.

D. L. Yarnell of the Bureau of Agricultural Engineering has just finished charting rates of precipitation and their frequencies for the 206 Weather Bureau Stations which have automatic rain gauges. The data cover periods of from 10 years up to 30 or more. Because a storm of great intensity usually extends over only a small area, Yarnell believes this study is sufficiently comprehensive for a reasonably accurate estimate as to how often a rain of a given intensity is likely to occur in any region of the United States.

The records for the station at Baltimore, Md., extend over a period of 38 years. The data imply that once in about five years, for a 5-minute period, Baltimore and vicinity can expect a rainfall at the rate of 6.2 inches an hour or 0.51 inch in 5 minutes; once in 10 years a rain at the rate of 6.82 inches an hour. The maximum amount of rain that ever fell at Baltimore, in the past 38 years, in a 5-minute period was 0.70 in. and for a 15-minute period 2.08 inch.

C.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

DETERMINATION OF BORIC OXIDE IN GLASS—A MODIFIED METHOD.

The routine determination of boric oxide in glass by the Chapin method is somewhat laborious, particularly for control purposes. The following method has been found satisfactory without material sacrifice in accuracy. Decompose 0.5 g. of glass by fusion with 1 g. of sodium carbonate. Digest the fusion with water, cool, acidify with hydrochloric acid (1 : 1) using para-nitro-phenol as indicator. Add 1 ml. strong hydrochloric acid, transfer to graduated cylinder, and make volume up to 25 ml. Add 25 ml. absolute alcohol and 50 ml. ethyl ether and shake thoroughly. Let the mixture stand until the ether layer is separated, transfer 50 ml. of the ether layer to a small flask, add two drops para-nitro-phenol and neutralize with half normal sodium hydroxide. Add 1 ml. phenolphthalein and additional sodium hydroxide to a definite pink, and then twice the last amount of sodium hydroxide in excess. Remove all the ether and alcohol, first by careful evaporation and finally by vigorous boiling. Cool, dilute to about 35 ml. and just acidify with half normal hydrochloric acid. Then proceed with the titration of boric oxide by the usual method.

The amount of boric oxide found in the 50 ml. aliquot must be computed to that in the total ether layer and the latter has been found to be 51.2 per cent. of the total boric oxide in the sample taken.

FRICTIONAL RESISTANCE OF STEEL AND BRASS IN SHRINK FITS.

The familiar property of expansion and contraction with rise and drop in temperature of metals is frequently utilized in binding a shaft or pin in a collar or socket by a shrink fit. With most metals, an oversize of the shaft of one or two one-thousandths of an inch per inch of diameter can be accommodated by heating the encircling member before the two are assembled. The converse method of contracting the pin

* Communicated by the Director.

or shaft by cooling it in liquid air or "dry ice" has received only brief mention, probably on account of the supposed commercial impracticability of the operation. The widespread availability of solid carbon dioxide as an easily transported refrigerant has put this method on a more practicable basis.

As described in a paper by W. H. Swanger at the June meeting of the American Society for Testing Materials, some work was recently completed at the Bureau on the axial slip or shrink fits made by cooling 1-inch diameter cylindrical pins to − 80° C. (− 112° F.) in dry ice and inserting them into rings which were about 0.0015 inch smaller than the pins when both were at room temperature. Three combinations of pin and ring were used; a brass pin and brass ring; a brass pin and steel ring; and a steel pin and steel ring. The maximum resistance to axial slip developed after a small amount of initial slip was 1430 lb./in.^{2} of contact area in the all-brass unit and 6050 lb./in.^{2} of contact area in the all-steel unit. The higher resistance in the steel unit was due chiefly to the fact that seizing between pin and ring occurred to such an extent that particles of steel were torn bodily from the pin. In the brass unit no such seizing occurred. The maximum resistance developed between the steel ring and brass pin was 2100 lb./in.^{2} of contact area, and the pin was visibly scored by the steel ring.

WEAR OF METALS.

A series of metal-to-metal wear tests of plain carbon steel has recently been completed in which were duplicated the testing conditions which led Fink, in Germany, to conclude that the presence of oxygen was essential to appreciable wear of metals. In the present series of tests, little or no difference was observed in the rates of wear of the steel whether tested in hydrogen, in nitrogen, or in air.

It was further observed that under conditions permitting the formation of a slight oxide film on the wearing surfaces the rate of wear was relatively low, whereas under conditions which did not permit oxide film formation the loss by wear was relatively very great. That is, oxide films may reduce the rate of metal-to-metal wear of steel appreciably while

the absence of any oxygen on the surface may result in excessive wear.

EFFECT OF SURFACE FINISH ON INITIAL CORROSION OF STEEL UNDER WATER.

It is well known that the surface of certain metals, such as iron, steel, aluminum, zinc, and copper, is coated with an oxide film, which is oftentimes invisible if it is thin and closely adherent to the metal. The existence of such coatings has been established by several investigators. Coatings of this kind may protect the metal from further oxidation by preventing the diffusion of more oxygen. On the other hand, they may cause irregularities in the results of corrosion tests, especially if there are discontinuities in the oxide film. Such defects may be: (*a*) breaks caused by the expansion or contraction of the metal or by its being bent or scratched; (*b*) "weak spots" produced by inclusions in the metal at the surface or (*c*) a breakdown of the oxide film in certain solutions.

The effect of an oxide film present on the surface of a low carbon steel finished with various abrasives upon the initial corrosion rate in water flowing at different velocities (5 to 225 ft. per minute) was studied recently at the Bureau. The results of this investigation were given in a paper, "Observations on Effect of Surface Finish on the Initial Corrosion of Steel Under Water" by Leo J. Waldron and Edward G. Groesbeck, at the meeting of the American Society for Testing Materials this month.

MICROSCOPIC EXAMINATION OF STEELS AT HIGH TEMPERATURES.

Several years ago an apparatus was developed at Harvard University by Dr. B. A. Rogers, now a Research Associate at the Bureau, which permitted the direct observation of the microstructure of iron at high temperatures, even up to the melting point of the metal. This apparatus is now being applied to the study of the austenitic grain size in carbon tool steel in one of the current research projects at the Bureau.

While it is possible to maintain a bright, clean surface on a specimen of pure iron in this high-temperature metallographic microscope by passing hydrogen through the inclosed

specimen chamber, it is not permissible to use hydrogen when the specimen under examination is carbon tool steel because of the rapid loss of carbon caused by the hydrogen. The examination of the carbon steels is being carried out in an atmosphere of mixed CO and CO_2 and in an atmosphere of helium.

"SHATTER" CRACKS IN RAILS.

Residual stress measurements have been completed on 8-inch lengths from each of two rails from the same ingot, one cooled normally after rolling and the other cooled slowly through the blue heat temperature range by a recently developed industrial process. Six longitudinal specimens, one-half inch square in cross section, were cut from the head, web, and base of each of the 8-inch pieces. The bars from the center of the head of the normally cooled rail increased in length when isolated from the head, whereas similar bars from the slowly cooled rail decreased in length. The head of the normally cooled rail contained numerous "shatter" cracks but the slowly cooled rail was free from "shatter."

The results are interpreted as indicating that tensile stresses are normally produced in the central part of the head of a rail as it cools, and that in the "shattered" rail these stresses were relieved when the "shatter" cracks formed.

INTERNAL STRESS IN AIRPLANE TIE RODS.

Recent tests in the Bureau's metallurgical division have indicated that internal stresses set up by rolling, in 1050 carbon steel tie rods for airplanes, can be very largely removed by heating 30 minutes at 600° to 700° F. This heat treatment does not lower the mechanical properties of the material very materially. In stainless steel tie rods, about 50 per cent. of the internal stress can be removed by heating 30 minutes at 900° F. After this treatment the mechanical properties are considerably higher than in the as-received condition.

After heating at a temperature of 900° to 1000° F., however, the resistance to corrosion as shown by the boiling-nitric-acid test descreases sharply. The temperature control in heat treatment of this material would therefore have to be

very carefully controlled to obtain an appreciable relief of internal stress without impairing the corrosion resistance. It is probable that the presence of titanium or columbium often recommended as a "stabilizer" in the stainless steel may allow higher temperatures to be used.

REVISED SIMPLIFIED PRACTICE RECOMMENDATION FOR TAPER ROLLER BEARINGS.

Comparison of the list of sizes of taper roller bearings which appeared in the original draft of Simplified Practice Recommendation R67, approved at a general conference held in 1927, with the revised Recommendation approved in 1933 and now available in printed form, shows the increasing use which industry has made of bearings of this type.

The original list, largely for automotive use, ranged in bore size from ⅝ inch to 4½ inches. While there were many sizes within these limits, differing by small fractions, the fact that the maximum size was 4½ inches, indicates that bearings of this type were limited in application.

The current revision, known as Simplified Practice Recommendation R67–33, is evidence of the increasing field of usefulness of anti-friction bearings. The acting chairman of the standing committee, Mr. E. Wooler, chief engineer of the Timken Roller Bearing Co., who also is vice-chairman of the ball and roller bearing division of the Society of Automotive Engineers' standards committee, advised that the steep angle bearings up to 12 inches bore, which are added in the revision, provide for naval requirements, and for general industrial and automotive purposes.

In addition to the steep angle taper bearings, conventional bearings are divided in three series, Light, Medium, Heavy. In the Light series the bore sizes range from 0.375 inch to 3.750 inches. In the Medium series the sizes range from 0.750 inch to 11.00 inches, and the Heavy series, range from 1.00 inch to 12 inches.

In transmitting the revised recommendation, Mr. Wooler made the following explanation:

"The extended sizes in the medium series up to 11" and in the heavy series up to 12" were carefully worked out to our present day knowledge but they are not ready yet for

standardization. However, their recommendation as simplified practice should assist considerably in making these bearings adaptable as a standard in the future. We already have a considerable number of large bore bearings and they will become as complicated as the original list of small bearings unless some such steps as the simplified practice scheme be adopted. The steep angle list is a selection of bearings in even increments from ¾" up to 12" bore which include all the large production series in this type, although the number of other steep angle bearings is not so large as the conventional bearings."

Copies of this Recommendation are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

SIMPLIFICATION OF INDUSTRIAL TRUCK AND TRAILER SOLID TIRES.

A proposed simplified practice recommendation covering industrial truck and trailer solid tires, cured-on-type, has been mailed to all interests in the industry by the division of simplified practice, of the Bureau, for their consideration and written approval.

This schedule, submitted by the standing committee of the industry in charge of the recommendation, if accepted, will be added to Simplified Practice Recommendation R103, Industrial Truck and Trailer Solid Tires, Pressed-on-Type, recently revised and made effective from June 1, 1934.

CONFERENCE OF STATE UTILITY COMMISSION ENGINEERS.

The twelfth annual conference of State utility commission engineers was held at the Bureau on May 31 and June 1. The conference was attended by engineers representing nine States and the District of Columbia. The program and arrangements for the conference were in charge of an executive committee consisting of the following utility commission engineers: Rudolph H. Nexsen, chief power engineer, Department of Public Service, New York, chairman, and Harry Barker, consulting engineer, Vermont, J. H. Wiley, chief engineer, North Dakota, P. L. Holland, chief engineer, Maryland and G. C. Boyer, engineer, Virginia.

The following papers and discussions were presented: Customer and consumption data—residential customers, R. E. Purucker, electrical engineer, Wisconsin; load factor in relation to rates, Russell G. Warner, electrical engineer, Connecticut; use of indexes or cost translators in appraisals, Harry Barker, consulting engineer, Vermont; cost of electrical distribution, C. E. Bennett, engineer, Federal Power Commission; the taxicab situation, P. L. Holland, chief engineer, Maryland, and discussion, A. S. Bates, superintendent, motor transport division, Connecticut; economies and improvements in rural telephone service, Harry Barker, Vermont; public utility accidents—causes and prevention, L. G. Krause, chief engineer, Pennsylvania; butane and propane gases, W. M. Black, assistant chief engineer, Pennsylvania.

The proceedings of the conference are not published. A small number of copies of the papers is available, and copies will be sent to commission engineers so long as the supply lasts, upon application to the secretary of the conference.

The following executive committee was elected to make all arrangements for the annual conference of 1935: Harry Barker, Vermont, chairman; P. L. Holland, Maryland, vice chairman; E. H. Morris, West Virginia; J. H. Wiley, North Dakota; L. G. Krause, Pennsylvania. J. Franklin Meyer, National Bureau of Standards, Washington, D. C., is the secretary of the conference.

CO_2, Plants, Quanta (Science Service).—Prof. B. M. DUGGAR of the University of Wisconsin has succeeded in demonstrating that to capture one molecule of carbon dioxide, the green cell of a water plant requires the energy of 32 quanta of sunlight. This value of the efficiency of the cell, Prof. Duggar says, is considerably less than heretofore reported by other investigators.

Meteorite Scars—Lake Beds (Science Service).—Prof. DOUGLAS JOHNSON of Columbia University, referring to the "bays" of the Carolina coastal plain, which are great elliptical depressions lately called the result of a monster bombardment with fragments of a comet in some past age, declares them to be nothing more than shallow lake beds with sandy rims raised by the wind. The longer axes of these elliptical "bays" are in general parallel, a logical consequence of splashing meteorites. Nevertheless, the Professor believes it all was caused by wind control. These lakes are now for the most part filled with bog deposits, thickly overgrown with vegetation.

Tinted Eyeglasses (Science Service).—According to DR. W. W. COBLENTZ of the U. S. Bureau of Standards not all tinted eyeglasses are equally efficient and apparently some are quite useless. Glare caused by strong sunlight reflected from a light-colored road is easily eliminated by dark lenses of shade 3. The dark yellow glasses sold at the corner drug store for fifty cents Dr. Coblentz designates as shade 1, a much lighter type. For ordinary use these are probably satisfactory, but anything lighter would scarcely help at all in cutting out glare. In all probability, the chief cause of "eye strain" and subsequent headache experienced in long-distance driving is a difference in refraction of the unaided eyes. Only properly fitted spectacle lenses will give relief.

Record Concrete Pour (Science Service).—A new all-time construction record for the pouring of concrete was established on the Boulder Dam project when in one day 10,462 cubic yards were mixed and placed in the forms. Nearly 9000 cubic yards went into the dam itself and the remainder into the Arizona spillway, diversion tunnels, intake towers, power house and tunnels of the penstock system.

THE FRANKLIN INSTITUTE

PRESENTATION OF THE COMPOSING STICK USED BY BENJAMIN FRANKLIN WHEN JOURNEYMAN PRINTER IN LONDON.

SPECIAL MEETING OF THE BOARD OF MANAGERS OF THE FRANKLIN INSTITUTE.

Meeting held 12:30 p.m., Thursday, June 14, 1934.

The meeting was called to order by the President, Mr. Nathan Hayward, who announced that this was a special meeting of the Board of Managers of The Franklin Institute and that the first order of business was to receive from Mr. Fry a gift which he had brought to us from London.

He then called upon Mr. Wilfred W. Fry who spoke as follows:

"*Mr. President, Mr. Secretary, and gentleman of the Board of Managers of The Franklin Institute:*

"During a recent visit to London it was my privilege to be guest at a luncheon given in the ancient home of the Worshipful Company of Stationers and Newspaper Makers—Stationers Hall. The occasion was sponsored by that veteran editor and publisher, Ralph D. Blumenfeld.

"Mr. Blumenfeld was born in Appleton, Wisconsin, and has retained his American citzenship though residing most of his life abroad. He is Deputy Master of the Worshipful Company, of which His Royal Highness the Prince of Wales is Master. Since retiring from the editorship of the *Daily Express*, Mr. Blumenfeld is devoting himself to promotion of better understanding between the two great English-speaking nations. This brief explanation is essential, as it was he who made possible both the occasion and the gift which I am about to present to you.

"At the close of my endeavor to give the assembled company a brief review of the then situation in America, Mr. Blumenfeld arose and advised the company as follows:

"'For many years this composing stick, used by Benjamin Franklin when a journeyman printer in London, has been one of the treasured relics of this company. By order of the Court of the company, at its meeting held two days since, I am directed to present it to Mr. Fry as a Trustee of The Franklin Institute of Philadelphia, where we trust it will remain as a token of friendship and good will.'

"Replying, I said:

"'Mr. Chairman, my Lords and gentlemen, I am well-nigh overwhelmed by this amazing gift. Speaking in behalf of my friend the distinguished President of The Franklin Institute, Mr. Nathan Hayward, I desire to extend hearty thanks to the Worshipful Company of Stationers and Newspaper Makers and to you, Mr. Chairman, for this priceless treasure and the sentiments of friendship and good will thus conveyed in such exquisite fashion. I am confident that this action on the part of the Worshipful Company of Stationers and Newspaper Makers will further cement the friendship of the two great English-speaking nations, bound together as we are by the ties of common blood, common speech, and common ideals; and I urge you, as I do my own countrymen, for the sake of our children and our children's children to be loath to differ and determined to understand, inasmuch as the things upon which we seem to differ are infinitesimal compared with those upon which we most certainly agree.'

"After this the meeting adjourned.

"And now, Mr. President, and gentlemen, I have great pleasure in completing my happy tryst with our British friends by turning over to you as a permanent relic for our museum this gracious gift."

The composing stick was then presented to Mr. Hayward by Mr. Fry. It was accepted by Mr. Hayward as President of The Franklin Institute as follows:

"*Mr. Fry:* By authority vested in me as President of The Franklin Institute, I accept this splendid gift and I assure you, Sir, that the Institute appreciates fully the graceful action of the Worshipful Company of Stationers and Newspaper Makers in presenting this valuable relic to us.

"This ancient composing stick which Franklin used when he worked as a young man in the printing offices of Palmer and Watts in London, will be placed in the museum, where it will not only bring back to the children who see it the early life of this great man but will arouse the keenest interest of all associated with the printing industry.

"But it is more than a mere relic—it is an emblem of good will between two great English-speaking countries. It is a gift to us in America from one of the oldest guilds in London, and it was the property of Benjamin Franklin, who probably more than any man of his time expressed this good feeling. Long before the revolution, Franklin, when he was in London, exerted every effort for peace. It was his thought that each of the British Colonies, scattered throughout the world, should become a self-governing unit, but at the same time an integral part of one great imperial whole. Thus, a century before its realization, Franklin conceived the British Commonwealth of Nations.

"As Sir Esme Howard said: 'Had his advice been taken when he was Colonial agent in London, it is more than probable that the violent breach with the Mother Country, which he never desired, would never have taken place.'

"Sir Esme continues:

"'Englishmen must certainly honour the memory of this man who, while loving the country from which his forefathers came, always battled for the just rights of his own compatriots, and, having struggled to preserve peace to the utmost of his power, after the die was cast did not hesitate to work as vigorously for the cause of his own country in war as he had previously laboured for the preservation of peace.'

"And so, Sir, we fully appreciate the responsibility it entails on us to care for this emblem of good will between the two great English-speaking countries of the world."

The President then requested authority to send the following cablegram:

"R. D. Blumenfeld,
The Express,
London, England.

At a Special Meeting of the Board of Managers of The Franklin Institute, called for the purpose of receiving The Franklin Composing Stick, Mr. Fry today turned over to us this priceless relic. The Board direct me on their behalf to convey to you and through you to the Worshipful Company of Stationers and Newspaper Makers grateful thanks and to advise that the composing stick will remain in perpetuity in our museum fittingly designated as a goodwill gift from your ancient guild."

On motion, the request of the President was unanimously adopted.

The President then invited all present to remain for a luncheon, which was served in the Board of Managers Room, and declared the meeting adjourned.

HOWARD MCCLENAHAN,
Secretary.

INFORMAL OPENING OF BENJAMIN FRANKLIN MEMORIAL CHAMBER AND UNVEILING OF TABLETS.

At half after eleven o'clock on the morning of Wednesday, May sixteenth, the lovely new Benjamin Franklin Memorial Chamber, which is entered from the central entrance of the new building on the Parkway, was informally opened. This chamber may be visited by the public at this time. It will not be formally dedicated to the honor of the great American whose name it bears until the statue which is now being prepared by James E. Fraser, noted sculptor, is completed. When the statue is completed the entire building will be formally dedicated. For the time being this lovely room will be open for all who may wish to visit it. The chamber was opened by the cutting of the ribbon barring the entrance. This ceremony was performed by John T. Windrim, the Architect.

Four tablets were unveiled, one in honor of Mr. Cyrus H. K. Curtis. This tablet reads as follows:

TO HONOR
CYRUS H. K. CURTIS
WITHOUT WHOSE VISION AND COURAGE
UNFLINCHING LEADERSHIP AND
GENEROUS PUBLIC SPIRIT
THIS
MEMORIAL TO BENJAMIN FRANKLIN
COULD NOT HAVE BEEN CREATED

This tablet was unveiled by Mrs. Edward Bok, daughter of Mr. Curtis.

Mr. Nathan Hayward, President of The Franklin Institute, of which Mr. Curtis was a long time member and also a member of its Board of Managers, spoke of the great debt which the Institute owed to Mr. Curtis and stated that his wonderful personality played a very important part in the life of the Benjamin Franklin Memorial, Inc., of which Mr. Curtis was the first President.

The second tablet was unveiled which read as follows:

THIS BUILDING IS THE CONSUMATION
OF THE JOINT EFFORTS OF
THE BENJAMIN FRANKLIN MEMORIAL, INC.
SPONSORED BY
THE POOR RICHARD CLUB
OF PHILADELPHIA
AND
THE FRANKLIN INSTITUTE
OF THE STATE OF PENNSYLVANIA

This tablet was unveiled by Mr. Louis W. Wheelock, former President and now second Vice President of the Poor

Richard Club, who spoke a few words regarding the coöperation between these two organizations.

The third tablet which was unveiled read as follows:

THE BENJAMIN FRANKLIN MEMORIAL

THIS TABLET IS ERECTED IN GRATITUDE FOR THE COOPERATION OF ELEVEN THOUSAND GOOD CITIZENS AND OF THE OFFICERS AND DIRECTORS OF THE ORGANIZATIONS WHICH WERE PRIVILEGED TO LABOR JOYOUSLY IN PAYING LASTING HONOR TO

BENJAMIN FRANKLIN

Here follows the names of the officers in charge at the time of the drive for funds in 1930. The organizations represented were the Benjamin Franklin Memorial, Incorporated, The Poor Richard Club and The Franklin Institute of the State of Pennsylvania.

This tablet was unveiled by Mr. A. Atwater Kent, President of the Benjamin Franklin Memorial, Inc., Mr. Kent was elected in January of this year to fill the vacancy made by the death of Mr. Cyrus H. K. Curtis. Mr. Kent repeated the legend at the top of the tablet and then pointed out that the Franklin Memorial was the effort of a great civic movement in which Philadelphians had shown themselves how to do a handsome thing most handsomely.

The fourth tablet to be unveiled was located in the graphic arts section. Dr. McClenahan, Director of the Institute, spoke as follows:

On behalf of The Franklin Institute I acknowledge with greatest gratification the gift symbolized by the tablet which will immediately be unveiled by Miss Virginia T. Kent, the daughter of Mr. and Mrs. A. Atwater Kent. This tablet was erected in grateful recognition of the gift which made possible

the museum of the graphic arts, one of the most important sections of the museum.

Miss Kent then unveiled the tablet which reads as follows:

THE
MUSEUM
OF
THE GRAPHIC ARTS
IS MADE AVAILABLE FOR THE
ENJOYMENT AND INSTRUCTION
OF THE PUBLIC THROUGH
THE MUNIFICENCE OF
A. ATWATER KENT
AND HIS WIFE
MABEL LUCAS KENT
1933

UNVEILING OF TABLET IN HONOR OF ARCHITECT

At a special meeting of the Board of Managers of The Franklin Institute held at twelve-thirty on Thursday, June fourteenth, at which time the Franklin composing stick which had been brought back from Europe by Mr. Wilfred W. Fry was formally presented to The Franklin Institute, the President of The Franklin Institute, Mr. Nathan Hayward, called upon the Secretary, Dr. Howard McClenahan, who spoke as follows:

"Mr. President, Members of the Board of Managers, Friends of the Institute: I have another very pleasant matter to bring to your attention.

"The Board of Managers of The Franklin Institute di-

rected that a tablet be erected in this room in recognition of the beautiful and artistic work and the helpful service of our architect, Mr. John Torrey Windrim. I therefore have pleasure in unveiling the tablet which reads:

IN HONOR OF

JOHN TORREY WINDRIM

ARCHITECT OF

THE BENJAMIN FRANKLIN

MEMORIAL

THE BOARD OF MANAGERS

OF THE FRANKLIN INSTITUTE

HAS CAUSED THIS TABLET

TO BE ERECTED

IN RECOGNITION OF HIS

PROFESSIONAL ABILITY AND

CULTURED TASTE

"I think it can be said of this building that, in the perfection of its proportions, in the durability of its structure and in the chaste beauty of its ornamentation, both exterior and interior, it has few equals, and no superiors, in the United States.

"Mr. Windrim, this tablet is erected by our Board of Managers as a grateful expression of appreciation of all that your help and your services have meant to us."

Mr. Windrim: "Mr. President, Dr. McClenahan, Members of the Board: I am very much touched and value highly this honor which you pay me. The loyal coöperation of the United Engineers and Constructors made it possible for us to erect this sterling structure. My heartfelt thanks."

The President then invited all present to remain for a luncheon which was served in the Board of Managers Room and declared the meeting adjourned.

John Torrey Windrim

Architect

The Benjamin Franklin Memorial

June 27, 1934

218, NO. 1303—9

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, June 13, 1934.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 13, 1934.

MR. CHARLES D. GALLOWAY *in the Chair.*

The following reports were presented for final action:

No. 2982: Method of, and Apparatus for, Dust Removal.

This report recommended the award of the Certificate of Merit to Mr. George S. Kelley, of New York City, "In consideration of the invention of an apparatus for the removing of the dust produced in the drilling of rock and the resulting lessening of the silicosis hazard."

No. 2983: Tri-Dimensional Vibrograph.

This report recommended the award of the John Price Wetherill Medal to Dr. James E. Shrader, of Drexel Hill, Pennsylvania, "In consideration of the design and construction of a portable and easily operated instrument for the investigation of vibrations, which affords simultaneous records of the vibrations in three mutually rectangular directions, and in which the inventor displayed much ingenuity."

No. 2986: Optical Lever and Optical Strain Gauge.

This report recommended the award of the John Price Wetherill Medal to Dr. Louis Bryant Tuckerman, of the United States Bureau of Standards, "In consideration of the fundamental improvements which he has introduced into the optical lever, making it an instrument of hitherto unrealized precision, and of his ingenious application of this lever to a strain gauge."

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, June 20, 1934.)

RESIDENT.

MR. PHILO T. FARNSWORTH, Vice-President in Charge of Research, Television Laboratories, Ltd., Inc., Chestnut Hill, Philadelphia, Pa.

DR. H. A. STEVENS HOWARTH, Vice-President and General Manager, Kingsbury Machine Works, Philadelphia, Pa. For mailing: Grant Avenue, Torresdale, Philadelphia, Pa.

DR. IBERT MELLAN, Research Chemist, Franklin Laboratories, Philadelphia, Pa.

MR. RICHARD L. SNYDER, Retired Engineer, Glassboro, N. J.

MR. JULIAN F. SPIEGEL, Mechanical Engineer, Kingsbury Machine Works, Philadelphia, Pa.

MR. NICHOLAS STEPHENOFF, Chief Engineer, International Pulverizing Company, Philadelphia, Pa. For mailing: 121 Main Street, Moorestown, N. J.

NON-RESIDENT.

MR. ERNST G. FISCHER, Retired, Washington, D. C.
MR. ROGER W. STAMM, Electrical Engineer, Pottstown, Pa.

STUDENT.

MR. SEYMOUR MOSES, Philadelphia, Pa.
MR. R. M. PEARL, Detroit, Michigan.

MUSEUM MEMBERSHIP:

FAMILY.

MR. F. ALGERNON EVANS, Awbury, East Washington Lane, Germantown, Philadelphia, Pa.
MR. CLARENCE A. WARDEN, Witherspoon Building, Philadelphia, Pa.
MRS. F. S. SKINNER, Ontario and C Streets, Philadelphia, Pa.

INDIVIDUAL.

MRS. CHARLES E. COXE, Malvern, Pa.
MR. S. F. SHARPLESS, 1919 Walnut Street, Philadelphia, Pa.

CHANGES OF ADDRESS.

MR. F. L. DUBOSQUE, 51 Bayview Avenue, Great Neck, New York.
MR. J. V. GIESLER, Box 104, Knoxville, Tenn.
MR. OTTO GOTTSCHALK, Galeria Guemes, Buenos Aires, Argentine.
MR. G. R. HALL, 80–24 Broadway, Elmhurst, Long Island, New York.
MR. W. N. JENNINGS, "Carycrest," Moylan, Rose Valley, Pa.
REAR ADMIRAL WM. S. SIMS, U.S.N., Marion, Massachusetts.
MR. CHARLES A. STANWICK, 49 Melrose Avenue, East Orange, N. J.
MR. H. A. TOULMIN, JR., 906 Harries Building, Dayton, Ohio.

NECROLOGY.

Mr. C. Francis Jenkins, Washington, D. C.

LIBRARY NOTES.

RECENT ADDITIONS.

ALT, HAROLD L. Air Conditioning Simplified. Theory, Practice and Marketing. Extracted from Articles Published in "Domestic Engineering" during 1932, 1933 and 1934.

American Gas Association. Proceedings. 1933.

American Institute of Mining and Metallurgical Engineers. Ore Deposits of the Western States. Lindgren Volume, First edition. Sponsored by the Rocky Mountain Fund. 1933.

American Institute of Steel Construction. Steel Construction. A Manual for Architects, Engineers and Fabricators of Buildings and Other Steel Structures. Second edition. 1934

Annual Review of Biochemistry. Edited by James Murray Luck. Volume 3. 1934.

ARNOLD, JOHN P., AND FRANK PENMAN. History of the Brewing Industry and Brewing Science in America. Prepared as Part of a Memorial to the Pioneers of American Brewing Science, Dr. John E. Siebel and Anton Schwarz. 1933.

BATAULT, ÉMILE. Le Mystère et le Paradoxe du Vol Animal. 1933.

Beilstein's Handbuch der organischen Chemie. Vierte Auflage. Die Literatur bis 1 Januar 1910 umfassend. Heraus gegeben von der Deutschen Chemischen Gesellschaft. Achtzehnter Band: Heterocyclische Reihe. 1934.

BERL, ERNST. Berl-LungeChemisch-technische Untersuchungsmethoden. Bd. 1. Achte, vollständig umgearbeitete und vermehrte Auflage. 1934.

BOURCART, E. Insecticides, Fungicides, and Weed Killers. A Practical Manual on the Diseases of Plants and their Remedies, for the Use of Manufacturing Chemists, Agriculturists, Arboriculturists and Horticulturists. Translated from the French and adapted to British standards and practice. Second English edition, revised and enlarged by Thomas R. Burton. 1926.

BULL, A. J. Photo-Engraving. 1934.

DICKSEE, C. B. Compression Ignition Engines. 1933.

DITTLER, R., AND OTHERS. Handwörterbuch der Naturwissenschaften. Bd. 2–4, 6–9. 1933.

FATH, EDWARD ARTHUR. The Elements of Astronomy. A Non-mathematical Textbook for Use as an Introduction to the Subject in Colleges, Universities, etc., and for the General Reader. Third edition. 1934.

FERMAT, PIERRE DE. Abhandlungen über Maxima und Minima (1629). Aus dem Lateinischen übersetz und mit Anmerkungen versehn von Max Miller. Ostwald's Klassiker der exakten Wissenschaften. Nr. 238. 1934.

FRIEND, J. NEWTON, Editor. A Text-Book of Inorganic Chemistry. Volume VI, Part 2: Phosphorus, by Edmund B. R. Prideaux. 1934.

GLAUERT, H. Wind Tunnel Interference on Wings, Bodies and Airscrews. Aeronautical Research Committee Reports and Memoranda No. 1556. 1933.

GRANT, HARRIS D. Simplified Mathematics for Accountants and Executives. Second edition. 1934.

HARROW, BENJAMIN, AND CARL P. SHERWIN. The Chemistry of the Hormones. 1934.

HELDT, P. M. Automotive Engines. Design, Production, Tests. Eighth edition of The Gasoline Motor. 1933.

Institute of British Foundrymen. Proceedings. Volume 26. 1932–1933.

Jahresbericht über die Leistungen der chemischen Technologie 1933. Bd. 79, 1 Teil. 1934.

KOCH, FREDERICK C. Practical Methods in Biochemistry. 1934.

LEHR, ERNST. Schwingungstechnik: ein Handbuch für Ingenieure. Erster Band: Grundlagen. Die Eigenschwingungen eingliedriger Systeme. 1930.

LYNAM, C. G. Growth and Movement in Portland Cement Concrete. 1934.

MCLACHLAN, N. W. Loud Speakers: Theory, Performance, Testing and Design. 1934.

MELLOR, J. W. A Comprehensive Treatise on Inorganic and Theoretical Chemistry. Volume XIII. 1934.

MUMFORD, LEWIS. Technics and Civilization. 1934.

NORD, F. F., UND R. WEIDENHAGEN. Ergebnisse der Enzymforschung. 3 Band. 1934.

Oil Burner Heating, with Special Emphasis on Subjects Related to General Automatic Heating. 1934.

RAPSON, E. T. A. Electrical Transmission and Distribution. 1933.

RITCHIE, PATRICK D. Asymmetric Synthesis and Asymmetric Induction. St. Andrews University Publication No. XXXVI. 1933.

Royal Society of London. Philosophical Transactions. Series A, Volume 232. 1934.

SPANNHAKE, WILHELM. Centrifugal Pumps, Turbines, and Propellers. Basic Theory and Characteristics. Authorized translation by John B. Drisko 1934.

SWANN, W. F. G. Architecture of the Universe. 1934.

TEMPLE, G., AND W. G. BICKLEY. Rayleigh's Principle and Its Applications to Engineering. The Theory and Practice of the Energy Method for the Approximate Determination of Critical Loads and Speeds. 1933.

THORPE, JOCELYN FIELD, AND REGINALD PATRICK LINSTEAD. The Synthetic Dyestuffs and the Intermediate Products from which They Are Derived. Being the seventh edition of "Cain and Thorpe" entirely rewritten and enlarged. 1933.

THORPE, JOCELYN FIELD, AND M. A. WHITELEY. Thorpe's Dictionary of Applied Chemistry. Supplement. Volume 1. 1934.

Van Nostrand's Chemical Annual. A Hand-Book of Useful Data for Analytical, Manufacturing, and Investigating Chemists, Chemical Engineers, and Students. Edited by John C. Olsen. Seventh Issue. 1934

VAUTHIER. Le Danger Aérièn et l'Avenir du Pays. 1930.

WAALS, J. D. V. D. Lehrbuch der Thermostatik das heisst des thermischen Gleichgewichtes materieller System. Bearbeitet von Ph. Kohnstamm. Erster Teil: Allgemeine Thermostatik. Zweiter Teil: Binäre Gemische. 1927.

BOOK REVIEWS.

CHEMICAL ECONOMICS, by Williams Haynes, 310 pages, illustrations, plates, 14 × 22 cms. New York, D. Van Nostrand Co., Inc., 1933. Price $3.25.

Williams Haynes as publisher of *Chemical Markets* is one well equipped to describe the chemical industry today as well as its beginnings of the day before yesterday. He does this under two heads, the Economic Foundations of Part One and the Historical Background under Part Two.

Innumerable illustrations are used in tracing the development of chemical processes and the extension of their use. Consider a simple one, the tanning of hide. There is on the table in a certain long room, which I know intimately, a lamp furnished with a parchment shade. To me this shade is interesting because of the plain song that some monk inked on the skin. Those in the monastery could read the notes at which I can only guess. They could entune the Latin verse

which is lost on me. They could join in a chant that may still echo through the broken arches of some deserted abbey. I have an affection for that shade. How beautifully it illustrates the very thing that is described under the head of modern tanning, page 16. Certainly chemistry was involved in the making of the ink. The monk who prepared the skin may not have employed all the materials used today, but he knew his fleshing, scudding, bating, tanning and finishing. At least that is how I interpret the light shed by that shade of plain song. My monk was a chemist.

Williams Haynes takes the hide story a bit further. He credits to a series of fortunate accidents, the skill our ancestors gathered so that they split hides for window panes which have been replaced by glass and, in the automobile, non-shatterable glass. Each step is a step in chemistry. I await that time not far distant when instead of a lamp, vestigial reminder of primitive ways, I shall have the cold light of a synthetic glow worm by which to read. I may have to part with the plain song shade or convert it into a suitable parchment jacket, but chemistry will make all of this possible.

Production of the chemicals of commerce are described in Chapter II. Supply and Demand, an ancient but imperative ratio in Chapter III, especially as it bears on Cost, Value and Price in Chapter IV, and finally Distribution in Chapter V. These chapters, to put it frankly, sound a bit dry. They are not. The author has a happy way of dealing with prosy facts that makes the account interesting. He pleads a bit for the sensitive chemical market, due to the workings of those amiable twins, Supply and Demand. I am not entirely convinced that the economics of the industry is as impeccable as described, but it is my lot to buy calcium chloride, when a lurking thought hints that the alkali makers have a corner on the road—for which the customer pays.

One very attractive feature of this book is the use of clever illustrations. One might say shadows on Salt, Alum Twilight, The Ammonia Battery or Cadavers, Containers, so unusual are the full page photographs of chemicals and apparatus. Of course any serious book of this kind teems with useful bits of information. The astute utilization of dung by the priest of Ammon which is memorialized in the word ammonia or the full name of I. G. (Interessen Gemeinschaft Farbenindustrie Aktiengesellschaft) and many other facts to reward the reader.

The obvious contribution of this book to the chemical arts is a summary review which the industry enjoys throughout the second part. Early American chemical history is described, clear in outline and rich in detail. The faces of men active through the years illumine the text. The impact of the world war and its tremendous effect on this country's chemical workings justify the informing chapter on this particular phase. But of slightly less interest is the chapter on cartels and consolidations, the intricacies of which must have been quite as baffling to the author as the labyrinth of Minos was to Theseus. Maybe he employed viscose line instead of the thread of Ariadne?

So through ways at home, cartels abroad and into the chemical future, Williams Haynes directs the steps of anyone interested in arts and artifices of chemistry. It reminds me of a time that I stood at the foot of a bed on which lay a very dear friend in delirium. She mistook me for the devil. With a weaving of the arms that made the room reel, she pointed in my direction and said with terrible earnestness "Now the magic begins."

H. W. ELKINTON.

COLLEGE PHYSICAL SCIENCE, AN ORIENTATION COURSE, by Paul McCorkle, Ph.D., Professor of Physics and Physical Science, State Teachers College, West Chester, Pa., and J. Arthur Lewis, M.A., Professor of Chemistry and Physical Science, State Teachers College, West Chester, Pa., 327 pages, illustrations, tables, 15 × 22 cms. Philadelphia, P. Blakiston's Son & Co., Inc. 1934. Price $2.00.

During the last decade, the great desirability of orientation courses for the college student has been receiving increased recognition by those responsible for the formulation of college curricula. Of all the branches of study open to the student, those of the sciences probably are least familiar at the time when important decisions are to be made. An orientation course in Physical Science would be of immense aid to numbers of puzzled college entrants in such events and it is with such an object that McCorkle and Lewis were induced to write this book. The authors point out that in any large group of students commencing their college work there will be found three types: Students who have had little or no science and will not pursue science further; students who will major in science; and students who are potential science teachers. This book is intended to contain many essential facts that will give all such students an adequate concept of the field embraced in the Physical Science category.

Of necessity, a book of such broad scope must be limited to a more or less general discussion of its topics. Such limitation has reacted favorably in producing a work unburdened by mathematical detail and hence eminently suited to the class of reader intended. Many a high-school student would enjoy a book of this nature. In it will be found the answers to numerous questions regarding natural phenomena and it should furnish excellent supplementary reading for various Science Club activities.

The authors begin with the macrocosmos and after discussing the sun, moon, stars, planets, comets, and meteors, come back to earth. There are sufficient interesting things concerning our earth to fill six chapters. For one example, it makes an excellent clock. As for its structure, is there anything like it in the whole universe? For those who discuss the weather and would like to do something about it, there is a very interesting chapter upon that very subject—the weather we mean.

Passing from "Winds and Weather" to "Water" leads the reader from geology to chemistry. Under this branch of science will be found sections on cleaning, textiles, cellulose and plastics. Physical subjects occupy the remainder of the space. Some of these are: Energy, Temperature, Heat, Atomic Radiations, Light, Sound, Electricity and Electrical Apparatus.

Most of the chapters are terminated with a number of problems and experiments highly instructive in nature. "College Physical Science" is meant primarily to teach, to guide teachers in teaching the subject, and so quite naturally the volume is the work of two teachers.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

The Chemistry of Petroleum Derivatives, by Carleton Ellis, 1285 pages, tables, 16 × 23.5 cms. New York, Chemical Catalog Co., Inc., 1934. Price $18.00.

Electrokinetic Phenomena and Their Application to Biology and Medicine, by Harold A. Abramson, 331 pages, tables, illustrations, 15.5 × 23.5 cms. New York, Chemical Catalog Co., Inc., 1934. Price $7.50.

Seeing and Human Welfare, by Matthew Luckiesh, 193 pages, plates, 14 × 21 cms. Baltimore, Williams & Wilkins Company, 1934. Price $2.50.

United States Bureau of Mines, Mineral Resources of the United States, 1931, part II, Nonmetals, 675 pages, tables, 15 × 23.5 cms. Washington, Government Printing Office, 1933. Price $1.00

Canada Department of Mines, Investigations in Ore Dressing and Metallurgy, 1932, 287 pages, tables, plates, 16.5 × 25 cms. Ottawa, King's Printer, 1934.

Bell Telephone Laboratories, Monographs, B–773, Electron Diffraction and the Imperfection of Crystal Surfaces, by L. H. Germer, 7 pages, illustrations, 15 × 23 cms. B–775, Effects of Rectifiers on System Wave Shape, by P. W. Blye and H. E. Kent, 23 pages, tables, plates, 15 × 23 cms. B–776, Investigation of Rail Impedances, by Howard M. Trueblood and George Wascheck, 27 pages, illustrations, plates, 15 × 23 cms. B–778, Joint Use of Poles with 6900 Volt Lines, by W. R. Bullard and D. H. Keys, 21 pages, illustrations, 15 × 23 cms. B–781, Physical Evaluation of Finishes, by A. E. Schuh and H. C. Theuerer, 16 pages, tables, 15 × 23 cms. B–782, Capillary Penetration of Fibrous Materials, by R. L. Peek, Jr., and D. A. McLean, 16 pages, illustrations, 15 × 23 cms. B–783, Open-Wire Crosstalk, by A. G. Chapman, 84 pages, illustrations, 15 × 23 cms. B–784, Auditory Perspective, A Symposium of Six Papers, 70 pages, illustrations, 15 × 23 cms. B–785, The Carbon Microphone, by F. S. Goucher, 32 pages, illustrations, 15 × 23 cms. New York, Bell Telephone Laboratories, 1934.

National Advisory Committee for Aeronautics, Technical Notes, No. 498, Wind-Tunnel Measurements of Air Loads on Split Flaps, by Carl J. Wenzinger, 8 pages, tables, illustrations, 20 × 26.5 cms. Washington, Committee, 1934.

CURRENT TOPICS.

That In-law Problem.—VINCENT PETRULLO, of the University of Pennsylvania Museum, reports that among the Yaruro Indians of Venezuela, a son-in-law is forbidden to talk to his mother-in-law. If in camp together, they sit facing opposite directions, and if there is need for conversation, a third person acts as go-between.

Compton's Cosmic Rays (*Items of Interest* (Science Service)).—After a two year world-wide survey of cosmic rays DR. ARTHUR H. COMPTON of the University of Chicago declares that "No evidence is found for either photons or electrons in the cosmic rays as they enter the earth's atmosphere." Intensity measurements of high altitudes seem to show that the rays capable of traversing the atmosphere consist chiefly of protons or positrons. The energies of the cosmic rays in remote space range from at least 2 to 40 billion volts and the spread of energy values may be even greater. Dr. Compton admits that low energy photons may constitute part of the absorbable cosmic rays observed in the upper atmosphere and that high energy photons may be part of the relatively weak component of very penetrating cosmic rays which is prominent at great depths. The very penetrating part may be partially made up of electrons so far as the Compton experiments are concerned.

Radiant Heat of Humans (*Items of Interest* (Science Service)).—Drs. DUBOIS and HARDY of the Russell Sage Institute describe a new device that quickly and easily takes the temperature of the surface of your body instead of the temperature inside of it. The device determines the surface temperature through radiant heat and actual bodily contact is not necessary. Determinations may be made through the clothing. More than half of the heat produced in the body is lost by radiation, the balance is lost chiefly by evaporation of moisture from the surface when you perspire.

C.

Keeping an Eye on the Milky Way (Science Service).—Most readers have heard before now about that very sensitive photo-electric cell-vacuum tube amplifier combination which can detect a candle flame at a distance of thirty miles. Not satisfied with this achievement DR. STEBBINS of the University of Wisconsin and DR.

WHITFORD of the Mount Wilson Observatory have tried combining this "electric eye" with Mt. Wilson's 100-inch reflector, thus being able to detect a candle at 3,000 miles—provided one is there. What the "eye" can do under these conditions is to measure the faint luminosity of the sky beyond the range of the photographic plate as well as classify star colors more precisely than the human eye or photography.

The first success of the new instrument used at Mt. Wilson was the detection of interstellar dust of the Milky Way through the reddening effect of the dust upon starlight. Just by measuring the colors of the stars in the Milky Way, Drs. Stebbins and Whitford were able to tell that the earth is at one side of the center of the great stellar system that comprises our Milky Way galaxy.

C.

Items of Interest (Science Service).—Drs. COMPTON and WOLLEN of the University of Chicago have exhibited lantern slides illustrating the appearance of atoms of helium, neon and argon should they be magnified 200,000,000 times. The pictures are intended to represent what the eye would see could it view these atoms through a powerful X-ray microscope. The method of preparation was to photograph a rapidly rotating template whose shape is calculated by a mathematical transformation of the scientists' measured values and of the X-rays scattered by helium, neon and argon gases.

Rendering Sun's Corona Visible.—A. M. SKELLET of the Bell Telephone Laboratories proposes to scan the image of the sky around the sun's disk with an "electric eye" photo-electric cell in much the same way that a television image is obtained. The scattering of direct sunlight produces a glare which prevents the seeing of the corona because it is many times brighter. However, from the photo-electric image it will be possible to sort out with electrical filters the high frequency components caused by the corona and then feed these into a television receiver which will reproduce the main features of the corona.

A 23-Year Weather Cycle.—DR. C. Y. ABBOT, secretary of the Smithsonian Institution, continues his studies on the 23-year cycle of weather ups and downs. He has found that weather data tended to show similarity when plotted in 23-year intervals. Similar plots of the thickness of annual water—deposited clay layers of glacial age, of tree-ring thickness during the past century, and of water-level in the Great Lakes during the same period, also indicate recurrence of weather features at approximate 23-year intervals.

C.

Bromine Content of Blood.—THEODORE FREDERIC DIXON (*Biochem. J.*, 1934, XXVIII, 48–51) analyzed 3 samples of normal human blood, and found their bromine content to be 0.89 1.20 and 1.73 milligrams, respectively, per 100 cc. of blood.

J. S. H.

Arsenic Content of American Cod Liver Oil.—ARTHUR D. HOLMES and ROE REMINGTON (*Ind. Eng. Chem.*, Ind. Ed., 1934, XXVI, 573–574) have determined the arsenic content of 20 samples of crude, medicinal, typical American cod liver oils. The arsenic content was from 1.4 to 5.1 parts per million, with an average value of 2.6 ± 0.13. The values are higher than those for common fruits and vegetables, decidedly lower than those for crustaceans, and of approximately the same magnitude as those for American marine fish and shell fish. The arsenic apparently is present in a form of low or no toxicity.

J. S. H.

Northern Lights.—L. VEGARD (*Science Progress*, 1934, XXVIII, 593–609) states that the northern lights or *aurorae polares* are the best means available for the study of the composition and physical state of the extreme upper layers of the atmosphere of the earth. Their spectrum reveals the presence of oxygen and nitrogen and of little or no helium and hydrogen. Apart from the formation of ozone in the upper atmosphere, the entire atmosphere is fairly constant in its chemical composition to its very top; as a matter of fact, a top layer of light gases does not exist.

J. S. H.

New Organic Detergents.—IVOR GRIFFITH (*Am. J. Pharm.* 1934, CVI, 176–177) calls attention to the alkali salts of certain aliphatic ester acids, and their use as detergents. The aliphatic group is the alkyl radical of a synthetic higher alcohol, such as lauryl, oleyl, cetyl, and stearyl. The acid is sulphuric acid. Thus sodium lauryl sulphate in aqueous solution is neutral in reaction, extremely active as an emulsifying and cleansing agent, acts in the presence of alkalies or acids (organic or inorganic) or salts of calcium, magnesium, and some heavy metals, and does not oxidize and become rancid. These detergents may find application in pharmaceutical chemistry.

J. S. H.

Arsenic Content of Irish Moss.—CHARLES H. LA WALL and JOSEPH W. E. HARRISON (*J. Am. Phar. Asso.*, 1934, XXIII, 308–309) have analyzed 13 samples of Irish moss (*Chondrus*), and found from 2 to 12 parts of arsenious oxide per million. The arsenic content therefore was in excess of the established tolerance (1.4 parts per million) for arsenic in food products; it was not uniform, even in samples from the same locality, and tended to be higher in the natural product than in the sulphur-bleached product.

J. S. H.

Benzoyl Peroxide in Flour.—Benzoyl peroxide is used extensively for bleaching flour. When used for that purpose it is readily detected in freshly milled flour, but is found with difficulty after the flour has aged for even a few days. Apparently it is converted into benzoic acid. The quantity of the peroxide added to the flour will produce between 20 and 34 parts of benzoic acid per million parts of flour. DOROTHY B. SCOTT (*J. Asso. Off. Agric. Chem.*, 1934, XVII, 302–304) has devised a method to detect the presence of this benzoic acid. The test is qualitative; approximately 25 per cent. of the benzoic acid is recovered. However the test is so delicate that less than 1 milligram of benzoic acid yields a positive reaction.

J. S. H.

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THE FRANKLIN INSTITUTE

Notice is hereby given that the Committee on Science and the Arts of THE FRANKLIN INSTITUTE will recommend to the Board of Managers the award of the

JOHN PRICE WETHERILL MEDAL

to

DR. LOUIS BRYANT TUCKERMAN

of

Washington, D. C.

"In consideration of the fundamental improvements which he has introduced into the optical lever, making it an instrument of hitherto unrealized precision, and of his ingenious application of this lever to a strain gauge,"

unless valid objection to the proposed award, based on evidence of lack of merit, is communicated, within one month of the date of this notice, to the Secretary of THE FRANKLIN INSTITUTE, Philadelphia, Pa.

HOWARD McCLENAHAN,
Secretary

Hall of the Institute
July, 1934

AWARDS BY THE INSTITUTE

The following awards are made by The Franklin Institute:

The Franklin Medal (Gold Medal).—This medal is awarded annually from the Franklin Medal Fund, founded January 1, 1914, by Samuel Insull, Esq., to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

The Elliott Cresson Medal (Gold Medal).—This medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

The Howard N. Potts Medal (Gold Medal).—This medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries; inventions or products of superior excellence or utilizing important principles.

The George R. Henderson Medal (Gold Medal).—This medal is to be awarded for meritorious inventions or discoveries in the field of railway engineering.

The John Price Wetherill Medal (Silver Medal).—This medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

The Edward Longstreth Medal (Silver Medal).—This medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes. In the event of an accumulation of the fund for medals beyond the sum of one hundred dollars, it is competent for the Committee on Science and the Arts to offer from such surplus a money premium for some special work on any mechanical or scientific subject that is considered of sufficient importance.

The Louis E. Levy Medal (Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

The Walton Clark Medal (Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

The Certificate of Merit.—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

The Boyden Premium.—Uriah A. Boyden, Esq., of Boston, Mass., has deposited with The Franklin Institute the sum of one thousand dollars, to be awarded as premium to "any resident of North America who shall determine by experiment whether all rays of light and other physical rays are or are not transmitted with the same velocity."

For further information relating to these awards apply to the Secretary of the Institute.